

Problem statement Consider the following improper integrals:

$$(1) \int_1^{\infty} \frac{\ln x}{x} dx; \quad (2) \int_1^{\infty} \frac{\ln x}{\sqrt{x}} dx \quad (3) \int_1^{\infty} \frac{\ln x}{x^3} dx.$$

- a) Graph the integrands. Determine which integrals are larger than the others for large x .
b) Do integrals (1) and (2) converge or diverge?

Hint Try a substitution to evaluate one of the integrals.

- c) Explain why your knowledge of integrals (1) and (2) does not help you decide whether integral (3) converges.
d) Explain why $\frac{\ln x}{x^3} < \frac{1}{x^2}$ for all $x > 1$. Use this to decide whether integral (3) converges.