

Problem statement A particle P moves in the plane. The *Rectangular Observer* computes the magnitude of the speed and acceleration of P by observing the x - and y -coordinates of P (as functions of time) and using these formulas: speed = $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ and acceleration = $\sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}$.

The other observer, the *Polar Observer*, finds it easier and more natural to measure the polar coordinates r and θ of P (as functions of time), using herself as the origin, of course. The *Polar Observer* computes $\frac{dr}{dt}$, $\frac{d\theta}{dt}$, $\frac{d^2r}{dt^2}$, etc. What formula should the *Polar Observer* use to compute the speed of the particle P ? Deduce your answer from the formula for speed displayed above and the relationships among x , y , r , and θ . Finally, if r and $\frac{d\theta}{dt}$ are constant, derive this formula valid for uniform circular motion: acceleration = (speed)²/ r .