

642.581 Problem Set 1

[This short set is meant to give you some idea of what to expect and some early feedback on writing. All solutions are short *once found*, though not necessarily easy *to find*. (My versions take a little more than half a page total, though I'm not expecting this.) So if you find yourself writing a lot, please think again.]

[Let's say all graphs are finite where not stated otherwise.]

1. Prove that every tree with maximum degree Δ has at least Δ leaves (vertices of degree 1).

[No long or talky solutions, please. This can be done in essentially one line.]

2. When does a graph G have a spanning subgraph with all degrees odd?

[General note: wording like this means you should answer and *justify*.]

3. For a connected $G = (V, E)$:

- (a) There is a partition $E = R \cup B$ with

$$|d_R(v) - d_B(v)| \leq 1 \quad \forall v \in V$$

unless

$$\text{all degrees in } G \text{ are even and } |E(G)| \text{ is odd.} \quad (1)$$

[Of course $d_R(v)$ is the number of edges of R containing v .]

- (b) If (1) holds then there is no such partition.