NAME: Solutions - March 9, 2009

Math 135 - Sections 16 - 18

Exam #1 Form A **February 25, 2010**

Instructions: (1) There are eight problems worth a total of 100 points. The number of points assigned to each problem is shown in parentheses after the problem number.

(2) Show your work. Now credit will be given for unsupported answers to problems requiring comptation. You may receive credit for partially correct work even if your final answer is incorrect.

(3) The last page of this booklet is a formula sheet

(4) You may use a calculator, but not a laptop computer or any device with a typewriter keyboard.

#1 (10 points) Find an equation for the tangent line to the graph of the equation

$$y = \frac{x}{x^2+3}$$

at the point where $x = -2$. You may use any method. Show your work.

When $x = -2$, $y = \frac{-2}{(-2)^2+3} = \frac{-2}{7}$. Thus the line goes through the point $(-2, -\frac{2}{7})$.

By the quotient rule $y' = \frac{1(x^2+3) - x(2x)}{(x^2+3)^2} = \frac{3-x^2}{(x^2+3)^2}$. Thus $y'(-2) = \frac{-1}{49}$ and this is the slope of the line. Then the point-slope form of the equation for the line is $y + \frac{2}{7} = \left(-\frac{1}{49}\right)(x+2)$

Please do not write in this space

#1: #2: #3: #4: #5

#6: #7 #8: TOTAL:

#2 (20 points) (a) Let $f(x) = x^2 + 3x - 7$. **Use the definition of the derivative** to find $f'(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 3(x+\Delta x) - 7 - (x^2 + 3x - 7)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3x + 3\Delta x - 7 - x^2 - 3x + 7}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 3\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2x + \Delta x + 3 = 2x + 3$$

(b) Let $g(x) = \frac{3}{2x+1}$. **Use the definition of the derivative** to find $g'(x)$.

$$g'(x) = \lim_{\Delta x \to 0} \frac{\frac{3}{2(x+\Delta x)+1} - \frac{3}{2x+1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{3(2x+1) - 3(2(x+\Delta x)+1)}{(2(x+\Delta x)+1)(2x+1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{6x + 3 - 6x - 6\Delta x - 3}{(2x + 2\Delta x + 1)(2x+1)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-6\Delta x}{(2x+2\Delta x+1)(2x+1)\Delta x} = \lim_{\Delta x \to 0} \frac{-6}{(2x+2\Delta x+1)(2x+1)}$$

$$= \frac{-6}{(2x+1)^2}$$

#3 (10 points) Find an equation for the line which passes through the midpoint of the line segment joining the points $(7, -3)$ and $(-1, 4)$ and is perpendicular to this line segment. Show your work.

The midpoint is $\left(\frac{7-1}{2}, \frac{-3+4}{2}\right) = \left(3, \frac{1}{2}\right)$

The slope of the line through the given points is $\frac{4-(-3)}{-1-7} = \frac{7}{-8}$. Then the slope of a line perpendicular to this line is $\frac{8}{7}$. Then the point-slope form of the equation for the given line is $y - \frac{1}{2} = \left(\frac{8}{7}\right)(x-3)$.

#4 (15 points) Find each of the following limits or explain why it does not exist. Indicate your method.

(a) $lim_{x \to 2}\left(\frac{x^2-4}{x^2+3x-10}\right)$. $= \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x+5)} = \lim_{x \to 2} \frac{x+2}{x+5} = \frac{4}{7}$

(b) $lim_{x \to 3}\left(\frac{x^2-4}{x^2+3x-10}\right)$. Since the limit of the denominator is non zero, thus is $\frac{\lim_{x \to 3} x^2-4}{\lim_{x \to 3} x^2+3x-10} = \frac{5}{8}$

(c) $lim_{x \to 4^-}\left(\frac{|4-x|}{3x-12}\right)$. Since $x < 4$, $4-x$ is positive, so $|4-x| = 4-x$. Thus the limit is $\lim_{x \to 4^-} \frac{4-x}{3(x-4)}$

$= \lim_{x \to 4^-}\left(-\frac{1}{3}\right) = -\frac{1}{3}$

(d) $lim_{x \to 1}\left(\frac{x-1}{|x-1|}\right)$.

$\lim_{x \to 1^-} \frac{x-1}{|x-1|} = -1$

$\lim_{x \to 1^+} \frac{x-1}{|x-1|} = 1$ Since these are not equal, the limit does not exist

(e) $lim_{x \to 0}\left(\frac{sin(3x)}{sin(7x)}\right)$.

$= \lim_{x \to 0} \frac{3\left(\frac{\sin(3x)}{3x}\right)}{7\left(\frac{\sin(7x)}{7x}\right)} = \frac{3}{7}$

#5 (16 points) Find the derivative of each of the following functions. You may use any method and do not have to simplify your answers. Show your work.

(a) $\frac{x^2+x+1}{x^3-7}$. By the quotient rule, this ~~is~~ has derivative

$$\frac{(2x+1)(x^3-7) - (3x^2)(x^2+x+1)}{(x^3-7)^2}$$

(b) $ln(3x^2+2)$.

$$\frac{6x}{3x^2+2}$$

(c) $2x^2\sqrt{x} + \sqrt{2x+1}$. This function is $2x^{\frac{5}{2}} + (2x+1)^{1/2}$

so the derivative is $2\left(\frac{5}{2}\right)x^{3/2} + \frac{1}{2}(2x+1)^{-1/2}(2)$

$= 5x^{3/2} + (2x+1)^{-1/2}$

(d) $cos(e^{5x^2})$. The derivative is

$$\left(-\sin\left(e^{5x^2}\right)\right)\left(e^{5x^2}\right)(10x)$$

#6 (9 points) Define a function $g(x)$ by $g(x) = x^2 - a$ if $x < 2$, $g(2) = b$ and $g(x) = x^3$ if $x > 2$. Suppose $g(x)$ is continuous at $x = 2$. What are a and b? Show your work.

If $g(x)$ is continuous at $x = 2$

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} x^2 - a = 4 - a$$

$$= g(2) = b$$

$$= \lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} x^3 = 8$$

Thus $b = 8$ and $4 - a = 8$ so $a = -4$.

#7 (10 points) A spherical balloon is being inflated is such a way that its radius increases at the constant rate of 10 centimeters per minute. Find the rate of change of the volume of the balloon when the radius is 30 cm. (Recall that a sphere of radius r has volume $\frac{4}{3}\pi r^3$.) Show your work.

We want $\frac{dV}{dt}$ when $r = 30$.

Now $\frac{dV}{dt} = \left(\frac{dV}{dr}\right)\left(\frac{dr}{dt}\right)$. Since $V = \frac{4}{3}\pi r^3$,

$\frac{dV}{dr} = 4\pi r^2$. We are told $\frac{dr}{dt} = 10$.

Thus $\frac{dV}{dt} = (4\pi r^2)10$. When $r = 30$ this

is $4\pi (30)^2 \cdot 10 = 36{,}000\pi$

#8 (10 points) Suppose $f(x)$ and $g(x)$ are two functions which are defined and differentiable for all real numbers. Suppose that

$f(0) = 2, f(1) = 1, f(2) = 1,$
$g(0) = 0, g(1) = 2, g(2) = 2,$
$f'(0) = -3, f'(1) = 7, f'(2) = -9,$
$g'(0) = 4, g'(1) = 2$, and $g'(2) = 6$.

Let $h(x) = f(g(x))$. Find $h(2)$ and $h'(1)$. Show your work.

$$h(2) = f(g(2)) = f(2) = 1$$

$$h'(1) = f'(g(1))g'(1)$$
$$= f'(2)g'(1)$$
$$= (-9)(2) = -18$$