NAME: SOLUTIONS - MARCH 9, 2010

Math 135 - Sections 16 - 18

Exam #1 Form B **February 25, 2010**

Instructions: (1) There are eight problems worth a total of 100 points. The number of points assigned to each problem is shown in parentheses after the problem number.

(2) Show your work. Now credit will be given for unsupported answers to problems requiring comptation. You may receive credit for partially correct work even if your final answer is incorrect.

(3) The last page of this booklet is a formula sheet

(4) You may use a calculator, but not a laptop computer or any device with a typewriter keyboard.

#1 (9 points) Define a function $g(x)$ by $g(x) = x^3 + a$ if $x < 2$, $g(2) = b$ and $g(x) = x^2$ if $x > 2$. Suppose $g(x)$ is continuous at $x = 2$. What are a and b? Show your work.

Sine $g(x)$ is continuous at $x=2$ we have

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} x^3 + a = 8 + a$$

$$= g(2) = b$$

$$= \lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} x^2 = 4$$

Thus $b = 4 = 8 + a$ and so $a = -4$

Please do not write in this space

#1:	#2:	#3:	#4:	#5
#6:	#7	#8:	TOTAL:	

#2 (20 points) (a) Let $f(x) = x^2 + 2x + 11$. **Use the definition of the derivative** to find $f'(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 2(x+\Delta x) + 11 - (x^2+2x+11)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x + 11 - x^2 - 2x - 11}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2x + \Delta x + 2 = 2x+2$$

(b) Let $g(x) = \frac{3}{4x-1}$. **Use the definition of the derivative** to find $g'(x)$.

$$g'(x) = \lim_{\Delta x \to 0} \frac{\frac{3}{4(x+\Delta x)-1} - \frac{3}{4x-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3(4x-1) - 3(4x+4\Delta x-1)}{(4x+4\Delta x-1)(4x-1)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{12x - 3 - 12x - 12\Delta x + 3}{(4x+4\Delta x-1)(4x-1)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-12\Delta x}{(4x+4\Delta x-1)(4x-1)\Delta x} = \lim_{\Delta x \to 0} \frac{-12}{(4x+4\Delta x-1)(4x-1)}$$

$$= \frac{-12}{(4x-1)^2}$$

#3 (10 points) Suppose $f(x)$ and $g(x)$ are two functions which are defined and differentiable for all real numbers. Suppose that

$f(0) = 2, f(1) = 1, f(2) = 1,$
$g(0) = 1, g(1) = 2, g(2) = 2,$
$f'(0) = -3, f'(1) = 7, f'(2) = -9,$
$g'(0) = 4, g'(1) = 2,$ and $g'(2) = 6.$

Let $h(x) = f(g(x))$. Find $h(1)$ and $h'(0)$. Show your work.

$$h(1) = f(g(1)) = f(2) = 1$$

$$h'(0) = f'(g(0))g'(0) = f'(1)g'(0)$$
$$= (7)(4) = 28$$

#4 (10 points) Find an equation for the line which passes through the midpoint of the line segment joining the points $(6, 5)$ and $(-3, 1)$ and is perpendicular to this line segment. Show your work.

The midpoint of the line segment is

$$\left(\frac{6-3}{2}, \frac{5+1}{2}\right) = \left(\frac{3}{2}, 3\right).$$

The slope of the line through the given points is $\dfrac{5-1}{6-(-3)} = \dfrac{4}{9}$. Thus the slope of a line perpendicular to this line is $-\dfrac{9}{4}$. Then the point-slope form of the equation for the desired line is

$$y - 3 = \left(-\frac{9}{4}\right)\left(x - \frac{3}{2}\right).$$

#5 (15 points) Find each of the following limits or explain why it does not exist. Indicate your method.

(a) $lim_{x \to 2}\ (\frac{x^2+3x-10}{x^2-4})$.

$$= \lim_{x \to 2} \frac{(x+5)(x-2)}{(x+2)(x-2)} = \lim_{x \to 2} \frac{x+5}{x+2} = \frac{7}{4}$$

(b) $lim_{x \to 3}\ (\frac{x^2+3x-10}{x^2-4})$.

Since the limit of the denominator is nonzero, this is

$$\frac{\lim_{x \to 3} (x^2+3x-10)}{\lim_{x \to 3} (x^2-4)} = \frac{9+9-10}{9-4} = \frac{8}{5}$$

(c) $lim_{x \to 3^-}\ (\frac{|3-x|}{4x-12})$.

When ~~Since~~ $x < 3$, ~~is~~ $3-x > 0$ so $|3-x| = 3-x$.

Thus the limit is $\lim_{x \to 3^-} \frac{3-x}{4(x-3)} = -\frac{1}{4}$.

(d) $lim_{x \to 1}\ (\frac{|x-1|}{x-1})$.

$$\lim_{x \to 1^-} \frac{|x-1|}{x-1} = -1$$

$$\lim_{x \to 1^+} \frac{|x-1|}{x-1} = 1$$

. As these are different, the limit does not exist.

(e) $lim_{x \to 0}\ (\frac{sin(4x)}{sin(5x)})$.

$$= \lim_{x \to 0} \frac{4\left(\frac{\sin 4x}{4x}\right)}{5\left(\frac{\sin 5x}{5x}\right)} = \frac{4}{5}$$

#6 (16 points) Find the derivative of each of the following functions. You may use any method and do not have to simplify your answers. Show your work.

(a) $\frac{x^2-x+2}{x^3+4}$. By the quotient rule, the derivative is

$$\frac{(2x-1)(x^3+4)-(x^2-x+2)(3x^2)}{(x^3+4)^2}$$

(b) $ln(2x^2+5)$.

$$\frac{4x}{2x^2+5}$$

(c) $2x^2\sqrt{x}+\sqrt{3x+1}$. This function is $2x^{5/2}+(3x+1)^{1/2}$ so the derivative is $5x^{3/2}+\frac{3}{2}(3x+1)^{-1/2}$

(d) $cos(e^{3x^2})$.

$$\left(-\sin(e^{3x^2})\right)\left(e^{3x^2}\right)\left(6x\right)$$

#7 (10 points) A spherical balloon is being inflated is such a way that its radius increases at the constant rate of 20 centimeters per minute. Find the rate of change of the volume of the balloon when the radius is 40 cm. (Recall that a sphere of radius r has volume $\frac{4}{3}\pi r^3$.) Show your work.

We want $\frac{dV}{dt}$ when $r = 40$.

Now $\frac{dV}{dt} = \left(\frac{dV}{dr}\right)\left(\frac{dr}{dt}\right)$. Since $V = \frac{4}{3}\pi r^3$

we have $\frac{dV}{dr} = 4\pi r^2$. We are given $\frac{dr}{dt} = 20$

Thus $\left.\frac{dV}{dt}\right|_{r=40} = \left.(4\pi r^2)(20)\right|_{r=40} = 4\pi(40)^2\cdot(20)$.

$= 128{,}000\pi$

#8 (10 points) Find an equation for the tangent line to the graph of the equation

$$y = \frac{x}{x^2+4}$$

at the point where $x = -1$. You may use any method. Show your work.

When $x = -1$, $y = \frac{-1}{5}$. Thus the tangent line goes through the point $(-1, -\frac{1}{5})$.

By the quotient rule, $y' = \frac{1(x^2+4) - x(2x)}{(x^2+4)^2}$

so $y'(-1) = \frac{3}{25}$. Then the point-slope form of the equation for the tangent line is

$$y + \frac{1}{5} = \left(\frac{3}{25}\right)(x+1)$$