

Math 135, Section C7

Review problems for Exam #2 - July 2, 2010

Exam #2 is on Monday, July 12, from 6:00 to 7:20.

A review session will be held on:

Saturday, July 10, 2:00 - 4:30 PM in HILL-525 (BUSCH)

#1 Find $\frac{dy}{dx}$ if $2x + e^{xy} = 0$.

#2 Find an equation of the tangent line to the graph of $x^2y - 2xy^3 = 0$ at the point $(2, 1)$.

#3 Find $\frac{dy}{dx}$ at the point $(2, 1)$ on the graph of $(x - y)^3 + y^2 = 2$.

#4 Find the derivative of the function $(2x + 1)^{(3x+1)}$ (where $x > 0$).

#5 If $y = 2\sqrt{x} - 9$ and $\frac{dy}{dt} = 5$ find $\frac{dx}{dt}$ when $x = 9$.

#6 One end of a rope is fastened to a boat and the other end is wound around a windlass located on a dock at a point 5 feet above the level of the boat. If the boat is drifting away from the dock at the rate of 7 feet/minute, how fast is the rope unwinding at the instant when the length of the rope is 13 feet?

#7 A car travels north from the city of Centralia at the rate of 30 miles per hour, starting at 11 AM. A truck travels east from Centralia at the rate of 45 miles per hour, starting at noon. How fast is the distance between the truck and the car changing at 1 PM?

#8 Find $d(x\sqrt{x^2 - 1})$.

#9 Use differentials to approximate $\sqrt{9.04}$.

#10 The radius of a circle has been measured as 15 inches, but there is an error of 0.05 inch in the measurement. Give an approximate value for the error in the computed area.

#11 For each of the following functions:

- (i) find all critical numbers;
- (ii) find the intervals where the function is increasing;
- (iii) find the intervals where the function is decreasing;
- (iv) determine whether each critical point is a relative maximum, a relative minimum, or neither;

(v) find the intervals where the graph of the function is concave up and the intervals where the graph of the function is concave down;

(vi) find all points of inflection;

(vii) find all horizontal and vertical asymptotes (realizing that there may be none);

(viii) sketch the graph of the function.

(a) $f(x) = x - x^2$;

(b) $f(x) = x^3 - 3x^2 + 3$;

(c) $f(x) = \frac{1}{3-x}$;

(d) $f(x) = \frac{x+1}{2-x}$;

#12 Sketch the graph of a function satisfying the following conditions:

$$\lim_{x \rightarrow -\infty} f(x) = 1,$$

$$\lim_{x \rightarrow \infty} f(x) = -1,$$

$$\lim_{x \rightarrow 1^-} f(x) = -\infty,$$

$$\lim_{x \rightarrow 1^+} f(x) = \infty,$$

$$f'(x) > 0 \text{ if } x < -1, \text{ or if } 3 < x,$$

$$f'(x) < 0 \text{ if } -1 < x < 1, \text{ or if } 1 < x < 3,$$

$$f''(x) > 0 \text{ if } x < -3 \text{ or if } 1 < x < 4,$$

$$f''(x) < 0 \text{ if } -3 < x < 1 \text{ or if } 4 < x.$$

#13 Find the absolute maximum and minimum values of the following functions on the given intervals. Give the values of x for which the absolute maximum and absolute minimum are attained.

(a) $f(x) = x^2 - 6x + 1$, on $[1, 4]$,

(b) $f(x) = \frac{x^3}{3} - x^2 + 1$ on $[-3, 3]$,

(c) $f(x) = |2x - 1|$ on $[0, 2\pi]$.

(d) $f(x) = \sin^2(x) + \cos(x)$ on $[0, \pi]$.

#14 Find the value of each of the following limits:

(a) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x}$,

(b) $\lim_{x \rightarrow 0} \frac{x - \sin(x)}{x^3}$,

(c) $\lim_{x \rightarrow 0} x^{-5} \ln(x)$,

(d) $\lim_{x \rightarrow \infty} \frac{\ln(\ln(x))}{x}$,

(e) $\lim_{x \rightarrow 0} \left(\frac{1}{\sin(3x)} - \frac{1}{3x} \right)$,

(f) $\lim_{x \rightarrow 0^+} \left(\frac{2\cos(x)}{\sin(2x)} - \frac{1}{x} \right)$.

#15 A cylindrical can is to have volume 108π cubic inches. If the material used to make the top and bottom of the can costs twice as much as the material used to make the sides, what are the dimensions of the can that is least expensive to produce.

#16 A rectangular box (without a top) is to be made from a sheet of cardboard with width 5 inches and length 8 inches by cutting squares of equal size out of each corner of the sheet of cardboard and folding up the resulting flaps to make the sides of the box. Find the dimensions of the box of largest possible volume.