

Math 135, Section C7 - Review problems for Final Exam #1 - July 18, 2010

There will be a review session on Tuesday, July 20, from 6:00 to 9:00 PM in Hill-525.

#1 Find all x such that $|x - 2| < \frac{1}{2}$ and express your answer in interval notation.

#2 Let L be the straight line through the points $(-1, 3)$ and $(4, 7)$.

(a) Find an equation of L .

(b) Find an equation of a line which is parallel to L and passes through the point $(-2, -3)$.

(c) Find an equation of a line which is perpendicular to L and passes through the point $(-1, 2)$.

#3 Write an equation of the circle with center $(-1, 2)$ and radius 4.

#4 The graph of the equation $x^2 + y^2 - 6x + 2y = 26$ is a circle. What are the center and radius of this circle?

#5 Suppose that $f(x) = \frac{2}{x}$ if $0 < x < 2$, $f(2) = a$, and $f(x) = x^2 + b$ if $x > 2$. Suppose further that $f(x)$ is continuous at $x = 2$. What are a and b ? Explain why, using the definition of continuity.

#6 Find each of the following limits or explain why the limit does not exist. Do not use l'Hopital's rule in solving this problem:

(a) $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^2 - 1}$;

(b) $\lim_{x \rightarrow 2} \frac{x^2 + 3x + 2}{x^2 - 1}$

(c) $\lim_{x \rightarrow 1^+} \frac{2x - 2}{|x - 1|}$

(d) $\lim_{x \rightarrow 1^-} \frac{2x - 2}{|x - 1|}$

(e) $\lim_{x \rightarrow 1} \frac{2x - 2}{|x - 1|}$

(f) $\lim_{x \rightarrow 2} \frac{1}{(x - 2)^2}$

$$(g) \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

#7 Use the definition of derivative to find each of the following derivatives. Do not use l'Hopital's rule in computing limits.

$$(a) f'(x) \text{ if } f(x) = -x^2 + 2x + 1$$

$$(b) g'(x) \text{ if } g(x) = \frac{3}{x - 2}$$

#8 In each part, find $f'(x)$ by any method:

$$(a) f(x) = x^6 + 2x\sqrt{x} - x + 1$$

$$(b) f(x) = \frac{2x^3 - 1}{x^2 + x + 1}$$

$$(c) f(x) = \cos\left(x + \frac{1}{x}\right)$$

$$(d) e^{\sin(\sqrt{x})}$$

$$(e) \sin(e^{x^3+2x})$$

$$(f) f(x) = \frac{\sin(x)}{e^{2x+3}}$$

$$(g) f(x) = \frac{\cos(x)}{x^2 + 1}$$

$$(h) f(x) = \ln(\sin(x) + \cos(x))$$

$$(i) f(x) = \ln(x^4 + x^2 + 2)$$

#9 A particle moves along the x -axis with position $s(t)$ and velocity $v(t)$ at time t . If its acceleration at time t is $-12t$, $s(0) = 0$ and $v(0) = 12$, what is:

(a) The particle's velocity at time t ?

(b) The particle's position at time t ?

(c)) The total distance traveled by the particle $t = 0$ and $t = 5$?

#10 Suppose $f(x)$ a function which is differentiable for all x . Suppose further that $f(1) =$

3, $f(3) = -6$, $f'(1) = -2$, $f'(3) = 4$. Let

$$h(x) = x^2 \cos(\pi f(x)).$$

Find $h'(1)$ and $h'(3)$.

#11 Let

$$f(x) = 5 - 3x, \text{ if } x < 1;$$

and

$$f(x) = ax^2 + b, \text{ if } x \geq 1.$$

Suppose $f(x)$ is differentiable at $x = 1$. What are a and b ? Why?

#12 Show that

$$e^x = 3 \cos(x)$$

for some x in the interval $[0, \pi/2]$.

#13 Find $\frac{dy}{dx}$ if $x \sin(y) + y \cos(x) = x$.

#14 Find an equation of the tangent line to the graph of $x^3 y^2 - \frac{2}{xy} = 7$ at the point $(2, 1)$.

#15 Find the derivative of the function $(\sin(x))^{e^x}$.

#16 A light is located on a pole that is 100 feet high. Another pole of the same height is located 50 feet away. A rock is dropped from the top of the second pole at time $t = 0$, so its height at time t will be $100 - 16t^2$ feet (where t is measured in seconds). The rock casts a shadow on the ground. How fast is the shadow moving at time $t = 2$ seconds?

#17 At noon ship A is 80 miles west of ship B. If ship A is sailing north at 15 miles per hour and ship B is sailing west at 20 miles per hour:

- (a) how fast is the distance between them changing at 2PM?
- (b) When is the distance between the ships a minimum?

#18 Use differentials to approximate $\sqrt{16.01}$.

#19 The radius of a sphere has been measured as 6 inches, but there is an error of 0.01 inch in the measurement. Give an approximate value for the error in the computed volume.

#20 For each of the following functions:

- (i) find all critical numbers;

- (ii) find the intervals where the function is increasing;
- (iii) find the intervals where the function is decreasing;
- (iv) determine whether each critical point is a relative maximum, a relative minimum, or neither;
- (v) find the intervals where the graph of the function is concave up and the intervals where the graph of the function is concave down;
- (vi) find all points of inflection;
- (vii) find all horizontal and vertical asymptotes (realizing that there may be none);
- (viii) sketch the graph of the function.

(a) $f(x) = -x^3 - 2x - 1$;

(b) $f(x) = \frac{x^3}{3} - 3x^2 + 3$;

(c) $f(x) = \frac{x+1}{x^2}$;

#21 Sketch the graph of a function satisfying the following conditions:

$$\lim_{x \rightarrow -\infty} f(x) = 2,$$

$$\lim_{x \rightarrow \infty} f(x) = -1,$$

$$\lim_{x \rightarrow -1^-} f(x) = \infty,$$

$$\lim_{x \rightarrow -1^+} f(x) = \infty,$$

$$f'(x) > 0 \text{ if } x < -1, \text{ or if } 1 < 3 < x,$$

$$f'(x) < 0 \text{ if } -1 < x < 1, \text{ or if } 3 < x,$$

$$f''(x) > 0 \text{ if } x < -1 \text{ or if } -1 < x < 2 \text{ or if } 4 < x$$

$$f''(x) < 0 \text{ if } 2 < x < 4.$$

#22 Find the absolute maximum and minimum values of the following functions on the given intervals. Give the values of x for which the absolute maximum and absolute minimum are attained.

(a) $f(x) = 2x^2 + 8x + 1$, on $[0, 4]$,

(b) $f(x) = \frac{x^3}{3} - 2x^2 + 4x + 1$ on $[-3, 3]$,

(c) $f(x) = 2|x| + |x - 2|$ on $[-3, 3]$.

(d) $f(x) = \sin(x) - x$ on $[\frac{-\pi}{2}, \frac{\pi}{2}]$.

#23 Find the value of each of the following limits:

(a) $\lim_{x \rightarrow 0} \frac{-1 + \cos(x)}{2 - x^2}$,

(b) $\lim_{x \rightarrow 0} \frac{e^{x^2} - 2e^x + 1}{e^{x^2} + 4e^x - 5}$,

(c) $\lim_{x \rightarrow 0^+} x^5 \ln(x)$,

(d) $\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x^4 - 1}}$

#24 A rectangular garden is to have an area of 100 square feet and is to be surrounded by a 2 foot wide path of crushed stone. The contractor wants to choose the dimensions of the garden so that the cost of the crushed stone is as small as possible. What should the dimensions of the garden be?

#25 Approximate

$$\int_2^5 x^2 dx$$

by a Riemann sum using 3 intervals and righthand endpoints.

#26 Suppose

$$\int_1^3 f(x) dx = 7,$$

$$\int_1^8 f(x) dx = 5,$$

$$\int_3^{12} g(x) dx = 4,$$

and

$$\int_8^{12} g(x) dx = 2.$$

Find

$$\int_3^8 2f(x) - g(x) dx.$$

#27 Find each of the following indefinite integrals:

(a) $\int 3x^4 - 2x^3 + 7 dx$

(b) $\int \frac{7}{x} dx$

(c) $\int e^x + \cos(x) dx$

$$(d) \int (6x^2 + 4x)e^{2x^3+2x^2-1} dx$$

$$(e) \int \frac{x^2 + \sin(3x)}{x^3 - \cos(3x)} dx$$

$$(f) \int (\sin(x) - \cos(x))e^{\sin(x)+\cos(x)} dx$$

#28 Find each of the following definite integrals:

$$(a) \int_0^3 x^2 - 6x + 2 dx$$

$$(b) \int_{-\pi}^{\pi} \sin\left(\frac{x}{2}\right) dx$$

$$(c) \int_1^3 \frac{x+1}{x^2+2x+4} dx$$

$$(d) \int_{\frac{\pi}{2}}^{\pi} \sin(x)e^{\cos(x)} dx$$

#29 Find

$$(a) \frac{d}{dx} \int_2^x e^{-t^2} dt$$

$$(b) \frac{d}{dx} \int_{-x^2}^{\sin(x)} e^{-t^2} dt.$$