Math 300 - Review problems for Exam #1 - February 12, 2009

#1 Suppose A and B are true while P and Q are false. State whether or not each of the following is true and justify your answer.
(a) $(A \wedge P) \Rightarrow (P \wedge Q)$;
(b) $(A \vee \sim Q \vee \sim B) \Rightarrow (P \vee \sim Q)$.

#2: Make truth tables for each of the following propositional forms:
(a) $(P \vee Q) \wedge (\sim P \vee \sim Q)$;
(b) $((P \wedge Q) \vee (P \wedge \sim R)) \vee (P \wedge R)$.

#3 Prove that $P \Leftrightarrow Q$ is equivalent to $(P \wedge Q) \vee (\sim P \wedge \sim Q)$.

#4 Is each of the following a tautology, a contadiction, or neither?
(a) $(P \vee \sim Q) \Rightarrow Q$
(b) $(P \wedge Q) \vee \sim (P \vee Q) \vee (P \Rightarrow Q) \vee (Q \Rightarrow P)$.

#5 Which of the following statements are true (where the universe is the set of all real numbers)? Why?
(a) $(\forall x)(\exists y)((x^2 + 1)y = 1)$;
(b) $(\exists x)(\forall y)((x^2 + 1)y = 1)$;
(c) $(\forall x)(\exists y)((x + 1)y = 1)$;
(d) $(\exists x)(\forall y)((x + 1)y = 1)$;
(e) $(\exists N)((N \textit{ is an integer}) \wedge (N > 0) \wedge (\frac{1}{N}) < .001))$;
(e) $(\exists N)(\forall M)((N \textit{ is an integer}) \wedge ((M > N) \Rightarrow (\frac{1}{M}) < .001))$;
(f) $(\exists M)(\forall N)((N \textit{ is an integer}) \wedge ((M > N) \Rightarrow (\frac{1}{M}) < .001))$;

#6 Prove each of the following:
(a) If n is an integer, the 24 divides $x(x + 1)(x + 2)(x + 3)$.
(b) For every natural number N and every nonzero real number r there is a natural number M such that for all natural numbers $m > M$

$$\frac{1}{m} < \frac{r}{N}.$$

$7 (a) Give a direct proof that if x is an even integer and y is an odd integer, then xy is an even integer.
(b) Give a proof by contradiction to show that if a and b are integers and ab is odd, then a and b are both odd.

#8 Let $A = \{1,2,3,4,5\}, B = \{2,4,6,8\}, C = (1,5)$, and $D =$ the set of natrual numbers. Find:

(a) $A \cap B$;
(b) $A \cup B$;
(c) $A \cap \tilde{C}$;
(d) $C \cap D$;
(e) the power set of $B \cap C$.
(f) the power set of $\emptyset$.

#9 Let A, B, C be sets. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

#10 Give an example of a nested family of sets $\{A_1, A_2, ...,\}$ such that
(a) $\cap_{i=1}^{\infty} A_i = (2,3]$;
(a) $\cap_{i=1}^{\infty} A_i = [2,\infty)$.

#11 Prove that $\sqrt{5}$ is irrational.