Math 300 - Review problems for Exam #2 - April 10, 2009
Review Session: Saturday, April 11, 2-4 PM in Hill-124

#1 Use mathematical induction to prove that 8 divides $5^{2n} - 1$ for every integer $n \geq 1$.

#2 Use the well ordering principle to show that if a, b are natural numbers then there exist integers $q, r \geq 0$ such that $a = qb + r$ and $0 \leq r < b$.

#3 Suppose $\overline{\overline{A}} = 33$, $\overline{\overline{B}} = 17$, and $\overline{\overline{A \cap B}} = 12$. Find $\overline{\overline{A \cup B}}$.

#4 Suppose $\overline{\overline{A}} = 11$.
(a) Find $\overline{\overline{\mathcal{P}(A)}}$.
(b) How many subsets $B \subseteq A$ satisfy $\overline{\overline{B}} = 4$?

#5 (a) State the definition of a relation from A to B.
(b) Suppose $\overline{\overline{A}} = 7$, $\overline{\overline{B}} = 5$. How many relations from A to B are there?

#6 Let $A = \{1, 2, 3, 4, 5\}$. For each of the following relations from A to A state whether or not it is an equivalence relation. If it is an equivalence relation give the corresponding partition of A.
(a) $R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,4), (5,5)\}$.
(b) $R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,4), (4,5), (5,5)\}$.

#7 (a) State the reflexive, symmetric, anti-symmetric, and transitive properties of a relation.
(b) State the definition of an equivalence relation, of a partial order, of a total order, and of a partition.

#8 For $0 \leq n \leq 6$, let $J_n = \{k \in \mathbf{Z} | 7 \ \ divides \ \ k - n\}$. Show that $\{J_0, ..., J_6\}$ is a partition of the integers and describe the corresponding equivalence relation.

#9 (a) Define a relation R on the integers by aRb if and only if either $a = b$ or $a + 2 < b$. Is R a partial order? Is it a total order?
(b) Define a relation S on the integers by aSb if and only a and b have the same parity (i.e., both are even or both are odd) and $a \leq b$. Is S a partial order? Is it a total order?

#10 State the definition of a function f from a set A to a set B. State the definition of the domain, codomain, and range of f.

#11 Let $A = \{1, 2, 3, 4\}, B = \{x, y, z\}$. Let f be the function from A to B defined by

$f(1) = z, f(2) = x, f(3) = y, f(4) = z$ and g be the function from B to A defined by $g(x) = 4, g(y) = 3, g(z) = 2$. Find $f \circ g$ and $g \circ f$. Is either of the functions $f \circ g, g \circ f$ one-to-one? Onto?

#12 Suppose f is a function from A to C an g is a function from B to C. When will $f \cup g$ be a function? Why?

#13 State the definition of: $lim_{n \to \infty} x_n = L$.

#4 (a) Show that $lim_{n \to \infty} \frac{n+1}{1-2n} = \frac{-1}{2}$.
(b) Show that $lim_{n \to \infty} \frac{2^n}{3^n} = 0$.
(c) Show that the sequence given by $x_n = (-1)^n (1 - \frac{1}{n})$ diverges.