Math 351
Workshop #3
September 19, 2007

#1 (a) Let R be a ring and let r be an element of R. Show that if there is a homomorphism α from $\mathbf{Z}_m$ to R with $\alpha([1]) = r$, then $mr = 0$.

(b) Let R be a ring and let r be an element of R satisfying $mr = 0$ and $r^2 = r$. Show that there is exactly one homomorphism α from $\mathbf{Z}_m$ to R with $\alpha([1]) = r$.

(c) Find all pairs of positive integers m and n such that there is a nonzero homomorphism from $\mathbf{Z}_m$ to $\mathbf{Z}_n$.

(d) For each of the pairs you have found in (c), find all homomorphisms from $\mathbf{Z}_m$ to $\mathbf{Z}_n$.

#2 Let R be a ring and define Z(R) (which is called the *center* of R) to be the set of all elements s in R such that rs = sr for every element r in R.

(a) Show that Z(R) is a subring of R.
(b) Show that if R is a ring with identity element 1, then 1 belongs to Z(R).
(c) Prove that $Z(M(\mathbf{R})) = \mathbf{R}I$ (where $\mathbf{R}$ denotes the field of real numbers and I denotes the (2 by 2) identity matrix).
(d) Let R and S be rings. Prove that $Z(R \times S)$ is isomorphic to $Z(R) \times Z(S)$.

#3 Show that if m and n are relatively prime positive integers, then $\mathbf{Z}_m \times \mathbf{Z}_n$ is isomorphic to $\mathbf{Z}_{mn}$. (See problem #39 of Section 3.3 for hints.)