MATH 354-03 **February 16, 2005**

Practice questions for exam #1

I will work these problems at a review session on Saturday, 2/19, 3-5 PM in ARC-328.(ARC-328 is in the Mathematics and Science Learning Center on the third floor of ARC.) Remember that the exam is in class on Monday, 2/21.

#1 A furniture manufacturer wishes to determine how many tables, chairs, desks, or bookcases he should make in order to optimize the use of his available resources. These products utilize two different types of lumber: pine and oak. The manufacturer has on hand 1,500 board feet of pine and 1,000 board feet of oak. He has 800 hours of his employees' time available for the entire job. His sales forecast plus his back orders require him to make at least 40 tables, 130 chairs, 30 desks and no more than 10 bookcases.

Each table requires 5 board feet of pine, 2 board feet of oak, and 3 hours of labor.

Each chair requires 1 board feet of pine, 3 board feet of oak, and 2 hours of labor.

Each desk requires 9 board feet of pine, 4 board feet of oak, and 5 hours of labor.

Each bookcase requires 12 board feet of pine, 1 board feet of oak, and 10 hours of labor.

The manufacturer makes a profit of $12 on a table, $5 on a chair, $15 on a desk, and $10 on a bookcase.

Set up a linear programming model of this situation. State explicitly what each of your variables (for example, $x_1, x_2, ...$) represents). DO NOT attempt to solve the resulting linear programming problem.

#2 A manufacturer has distribution centers located in Atlanta (A), Chicago (C), and New York (NY). These centers have available 40, 20, and 40 units of his product, respectively. His retail outlets require the following number of units: Cleveland (CL)- 25; Louisville (L) - 10; Memphis (M)- 20; Pittsburgh (P)- 30; and Richmond (R)- 15. The shipping cost per unit in dollars between each center and outlet is given in the following table:

	C	L	M	P	R
A	55	30	40	50	40
C	35	30	100	45	60
NY	40	60	95	35	30

Set up a linear programming model of this situation. State explicitly what each of your variables (for example, $x_1, x_2, ...$) represents. DO NOT attempt to solve the resulting linear programming problem.

#3 Convert the following linear programming problem into (a) standard form, and (b) canonical form.

minimize: $-3x_1 + 2x_2 + x_4$
subject to:
$x_1 + x_2 \geq 5 - x_1 - x_3 + 2x_4$
$x_2 + 3x_4 = 5$
$x_1, x_2 \geq 0, x_3 \leq 0, x_4$ unconstrained.

#4 Consider the linear programming problem
Maximize: $x + y$
Subject to:
$-x + y \leq 2$
$2x + y \leq 6$
$x + 2y \leq 6$
$x, y \geq 0.$

Sketch the feasible region and the lines with equations $x+y = 2, x+y = 3, x + y = 4, x + y = 5, x + y = 6$. Find the optimal solution and explain how you find it.

#5 Consider the linear programming problem
Maximize: $x + y$
Subject to:
$-2x + y \leq 2$

$x - 2y \leq 2$
$3x + 5y \geq 15$
$x, y \geq 0.$

Sketch the feasible region. Does this problem have an optimal solution? Why or why not?

#6 Consider the linear programming problem
Maximize: $x + y$
Subject to:
$2x + y \leq 6$
$x + 2y \leq 6$
$2x + 3y \geq 24$
$x, y \geq 0$. Sketch the feasible region. Does this problem have an optimal solution? Why or why not?

#7 State the definitions of the following terms:
(a) feasible region of a linear programming problem
(b) convex set in $\mathbf{R}^n$
(c) extreme point of a convex set

#8 Let $\mathbf{x}$ and $\mathbf{y}$ be vectors in $\mathbf{R}^n$. Describe the line segment joining $\mathbf{x}$ and $\mathbf{y}$.

#9 Prove that the feasible region of a linear programming problem is convex.

#10 (a) Find an optimal solution (if there is one) to the following linear programming problem using the simplex method.
Maximize: $4x_1 + 3x_2 + 6x_3$
Subject to:
$3x_1 - 4x_2 - 6x_3 \leq 18$
$-2x_1 - x_2 + 2x_3 \leq 12$
$x_1 + 3x_2 + 2x_3 \leq 1$
$x_1, x_2, x_3 \geq 0.$

(b) Find an optimal solution (if there is one) to the following linear programming problem using the simplex method. (Note that only one coefficient has been changed from the problem in (a).)

Maximize: $4x_1 + 3x_2 + 6x_3$
Subject to:
$3x_1 - 4x_2 - 6x_3 \leq 18$
$-2x_1 - x_2 + 2x_3 \leq 12$
$-x_1 + 3x_2 + 2x_3 \leq 1$
$x_1, x_2, x_3 \geq 0.$

#11 In each part, find an optimal solution (if there is one) to the following linear programming problem using the two-phase simplex method (or the big M method).

(a) Maximize: $x_1 + x_2$
Subject to:
$-x_1 - x_2 + x_3 + x_4 = 2$
$-4x_1 - x_2 + 2x_3 + 3x_4 = 5$
$x_1, x_2, x_3, x_4 \geq 0$

(b) Maximize: $x_1 + x_2$
Subject to:
$2x_1 + x_2 + x_4 = 6$
$3x_2 + x_3 + x_5 = 8$
$3x_1 + 6x_2 + 2x_3 + x_4 + x_5 = 20$
$x_1, x_2, x_3, x_4, x_5 \geq 0$