MATH 354-03 **May 1, 2005**

Practice questions for Final Exam

I will work these problems at a review session on Tuesday, May 3, 2-5 PM in ARC-324 and again at a review session on Saturday, May 7, 2-5 PM in ARC-324. (ARC-324 is in the Mathematics and Science Learning Center on the third floor of ARC.) Remember that the exam is in on Monday, May 8 from 8:00 to 11:00 AM in the usual classroom.

#1 A lawn products company has 80 tons of nitrate and 50 tons of phosphate to use in producing three types of fertilizer. "Regular lawn" fertilizer requires 4 tons of nitrate and 2 tons of phosphate per 1000 bags. "Super lawn" fertilizer requires 4 tons of nitrate and 3 tons of phosphate per 1000 bags. "Garden" fertilizer requires 2 tons of nitrate and 2 tons of phosphate per 1000 bags. The profit per 1000 bags of fertilizer is \$300 for "regular lawn" fertilizer, \$500 for "super lawn" fertilizer, and \$400 for "garden" fertilizer.

(a) Set up a linear programming model of this situation. State explicitly what each of your variables (for example, $x_1, x_2, ...$) represents).

(b) Use the simplex method to find an optimal solution to this problem.

#2 A manufacturer has distribution centers located in Atlanta (A), Chicago (C), and New York (NY). These centers have available 40, 20, and 40 units of his product, respectively. His retail outlets require the following number of units: Cleveland (CL)- 25; Louisville (L) - 10; Memphis (M)- 20; Pittsburgh (P)- 30; and Richmond (R)- 15. The shipping cost per unit in dollars between each center and outlet is given in the following table:

	C	L	M	P	R
A	55	30	40	50	40
C	35	30	100	45	60
NY	40	60	95	35	30

(a) Set up a linear programming model of this situation. State explicitly what each of your variables (for example, $x_1, x_2, ...$) represents.

(b) Use the algorithm for the transportation problem to find an optimal solution.

#3 Problem #1 of Section 4.1.

#4 In each part, sketch the feasible region, find all extreme points of the feasible region, and sketch lines with equation $z = k$ for several values of k. Explain, on the basis of your sketch, whether the problem is feasible or not and whether or not it has an optimal solution. If it has an optimal solution, find it.

(a) Maximize $z = x_1 + 3x_2$
subject to
$x_1 + 2x_2 \leq 3$
$2x_1 + x_2 \leq 3$
$4x_1 + 3x_2 \geq 12$
$x_1, x_2 \geq 0.$

(b) Maximize $z = x_1 + 3x_2$
subject to
$-x_1 + x_2 \leq 1$
$2x_1 - x_2 \geq 2$
$x_1, x_2 \geq 0.$

(c) Maximize $z = x_1 + 3x_2$
subject to
$-x_1 + x_2 \leq 1$
$2x_1 - x_2 \leq 2$
$x_1, x_2 \geq 0.$

#5 Use the two-phase simplex method to solve each of the following linear programming problems.

(a) Maximize $z = x_1 + x_2$
subject to
$2x_1 - 3x_2 \geq 6$

$-x_1 + 2x_2 \geq 4$
$x_1, x_2 \geq 0.$

(b) Maximize $z = -x_1 - x_2$
subject to
$2x_1 - 3x_2 \geq 6$
$-x_1 + 2x_2 \geq 4$
$x_1, x_2 \geq 0.$

(c) Maximize $z = 2x_1 + x_2 + x_3$
subject to
$x_1 x_2 + x_3 = 3$
$x_1 + 2x_2 - 2x_3 = 5$
$x_1, x_2, x_3 \geq 0.$

#6 In the matrix below, the (i, j) entry represents the capacity of the (directed) arc from node i to node j in a network. Use the labeling algorithm to find the maximal flow from the source (node 1) to the sink (node 10). Also find a minimal cut.

$$\begin{bmatrix} 0 & 7 & 6 & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

#7 Convert the following linear programming problem to standard form. Then write the dual problem (using unrestricted variables and equalities where appropriate).

Minimize: $x_1 - 3x_2 + 4x_3$
Subject to
$x_1 - 2x_2 + x_3 = 7$
$2x_1 + 5x_2 + 3x_3 \geq 5$
$x_1, x_2 \geq 0, x_3$ unrestricted.

#8 Consider the linear programming problem:

Maximize: $4x_1 + 3x_2 + 6x_3$
Subject to:
$3x_1 - 4x_2 - 6x_3 \leq 18$
$-2x_1 - x_2 + 2x_3 \leq 12$
$x_1 + 3x_2 + 2x_3 \leq 1$
$x_1, x_2, x_3 \geq 0.$

Use the revised simplex method to solve this problem, giving the values of B^{-1} and $\mathbf{x}_B$ at each step and computing the $z_j - c_j$ from this information at each step.

#9 Solve problem #7 of Section 4.2 using
(a) the cutting plane method;
(b) the branch and bound method.

#10 Solve the assignment problem with matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 7 & 2 & 4 & 6 \\ 1 & 4 & 3 & 2 & 9 & 4 \\ 2 & 1 & 1 & 3 & 5 & 7 \\ 8 & 6 & 2 & 4 & 9 & 3 \\ 5 & 5 & 7 & 8 & 4 & 2 \end{bmatrix}.$$

#11 State and prove the Weak Duality Theorem.

#12 Consider the linear programming problem in standard form:

Maximize $\mathbf{c}^T\mathbf{x}$
subject to
$A\mathbf{x} \leq \mathbf{b}$
$\mathbf{x} \geq \mathbf{0}$.

State and prove the principle of complementary slackness relating the optimal solutions of this problem and the dual problem.

#13 Explain how the v_i and w_j used in the solution of the transportation problem are related to the dual problem and why they may be used to compute the $z_j - c_j$.

$14 Explain what changes can be made in the matrix for an assigment problem without changing the optimal solution and why these changes can be made.

#15 Consider the linear programming problem
Maximize: $x_1 + x_2 + 2x_3 + x_4$
subject to
$-11x_1 - 3x_3 + 2x_4 \leq 1$
$7x_1 + x_2 + 2x_3 - x_4 \leq 1$
$x_1, x_2, x_3, x_4 \geq 0$.

(a) Find the optimal solution.

(b) Find all values of Δc_3 such that the optimal solution $\mathbf{x}_B$ remains unchanged if the coefficient of x_3 in the objective function is replaced by $2 + \Delta c_3$.

(c) Suppose the constant on the right hand side of the first constraint is replaced by -2. Use your final tableau from part (a) and the dual simplex method to find the optimal solution to the new problem.