Math 428 - Review problems for Exam #1 - October 5, 2009

#1 G is a connected graph of order n in which every trail is a path. What is the size of G? Why?

#2 Is each of the following sequences graphical? If yes, give an example; if no, give a reason.

(a) (6,5,3,2,2,1,1,1)

(b) (7,6,5,4,4,4,3)

(c) (7,6,5,4,4,3,2,1)

(d) (7,6,6,5,4,3,2,1)

#3 Show that if G is a disconnected graph containing exactly two odd vertices, then these odd vertices must be in the same component of G.

#4 What is the smallest possible order of a 3-regular graph that contains the following graph? Find a graph of this order.
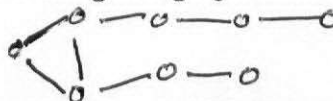
#5 Let G be a connected graph, T be a spanning tree in G and e an edge of G that is not in T. Show that $T + e$ contains a unique cycle.

#6 True or false: Any graph or order n with degree sequence $(2, 2, ..., 2, 1, 1)$ is isomorphic to P_n. Explain.
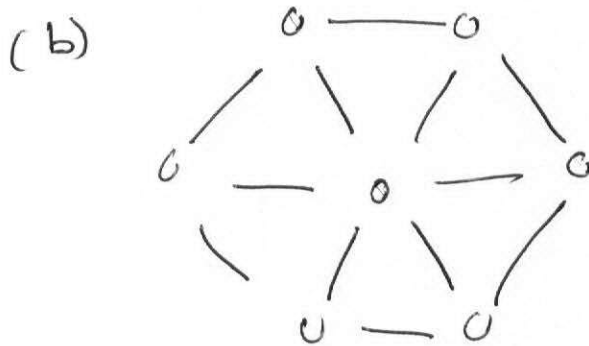
#7 True or false: Any 2-regular connected graph of order n is isomorphic to C_n. Explain.

#8 Draw:

(a) $2P_3 \cup 3C_4 \cup K_5$

(b) $\overline{C}_5$

(c) $C_3 + C_4$

(d) $C_3 \times C_4$

1

2

#9 Which pairs of the following graphs are isomorphic? Give reasons.
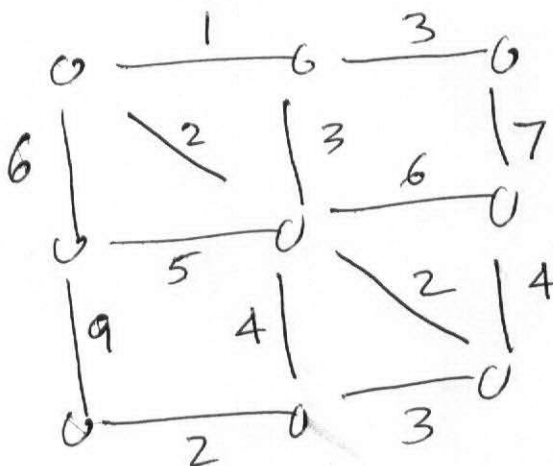
(a) $K_{4,3}$

(b)

(c)

(d)

#10 Find a minimal spanning tree for each of the following weighted graphs by using Kruskal's algorithm and also by using Prim's algorithm. In each case show the order in which edges are chosen.

(a)

(b)