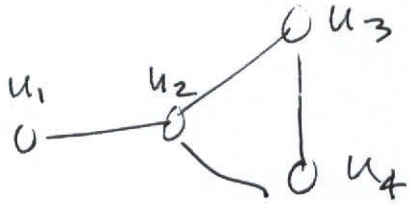
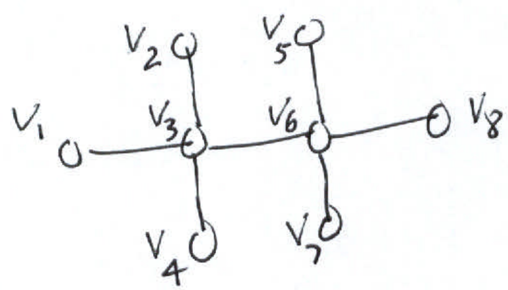
Math 428 - Review problems for exam #2

#1 In each of the following graphs find all cut-vertices and all bridges.

a)

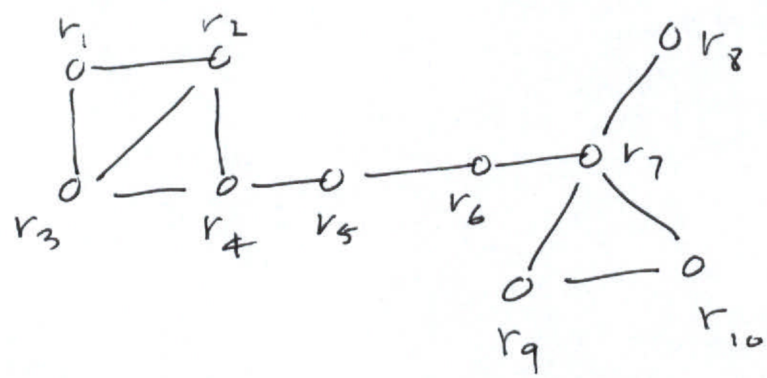


b)



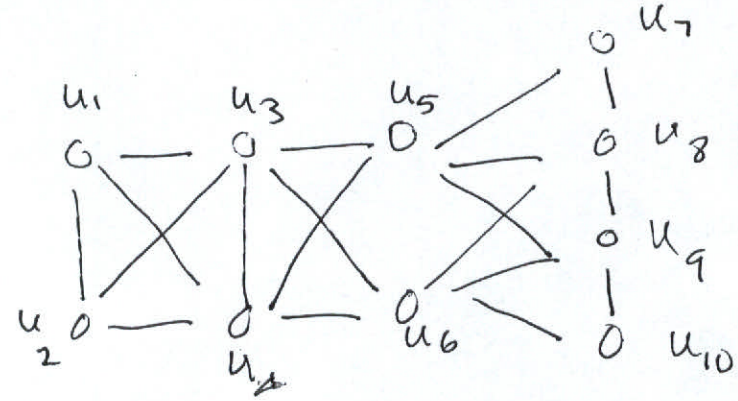
c)

#2 Find all blocks of

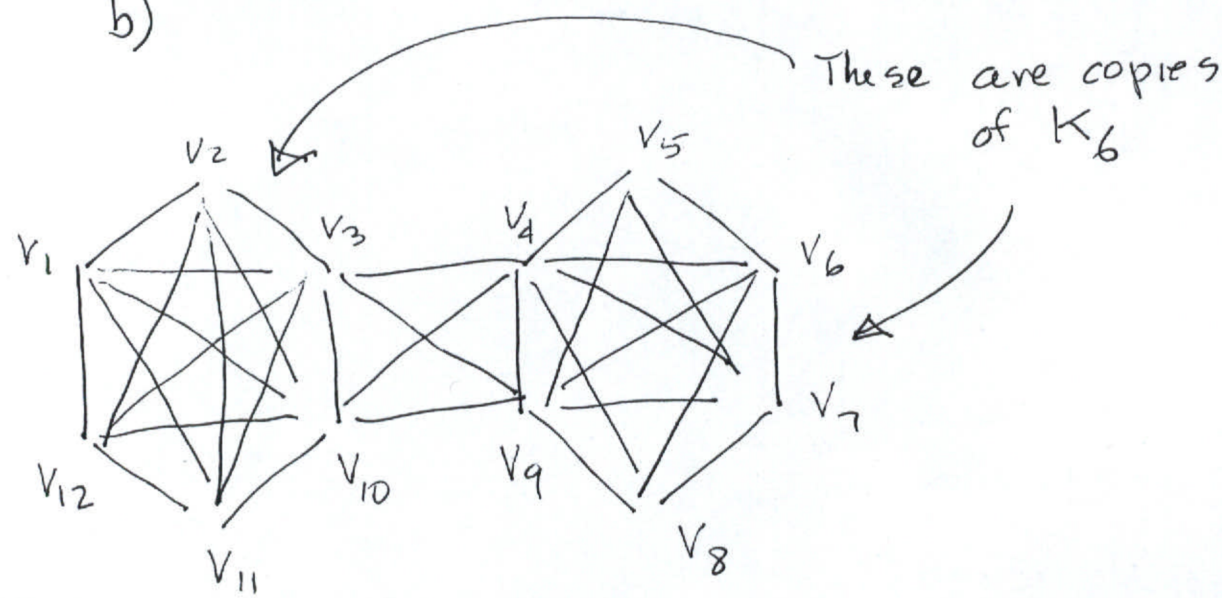


#3 Find a minimum vertex cut and a minimum edge cut of each of the following graphs

a)



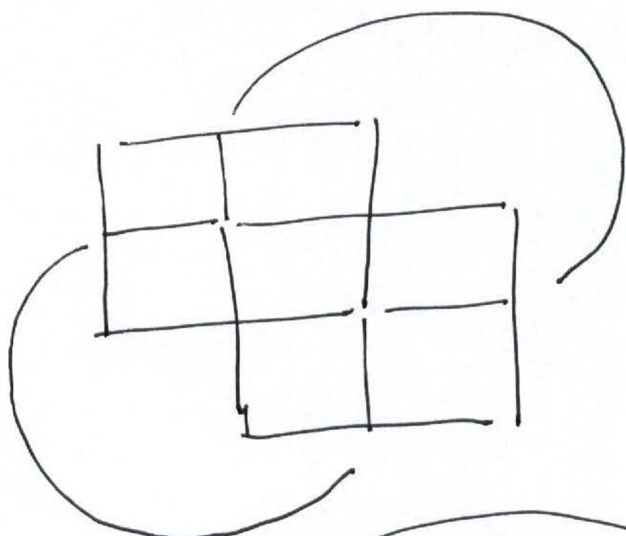
b)



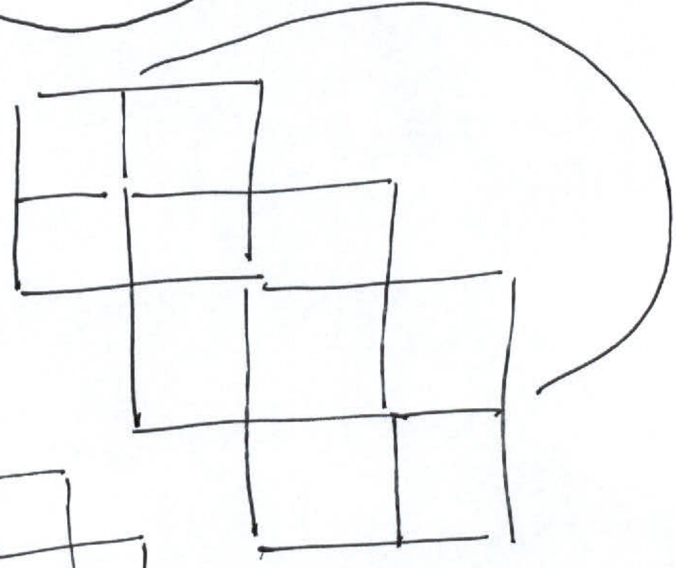
#4 Does the graph G_i (for $i = 1, 2, 3$) contain an Eulerian circuit? Why or why not? If it contains an Eulerian circuit find one.

Does the graph G_i (for $i = 1, 2, 3$) contain an Eulerian trail? Why or why not? If it contains an Eulerian trail find one.

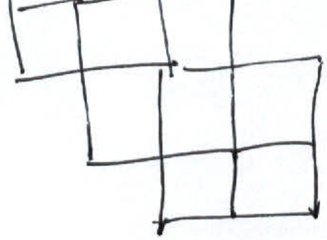
a) G_1



b) G_2

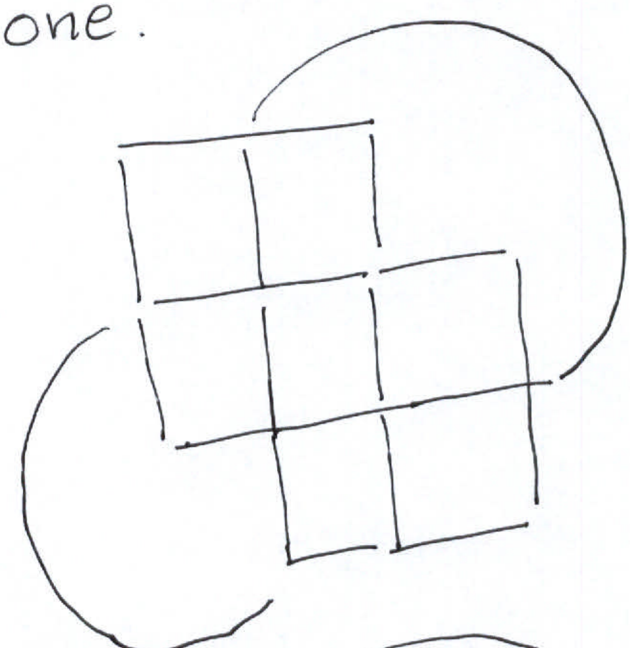


c) G_3

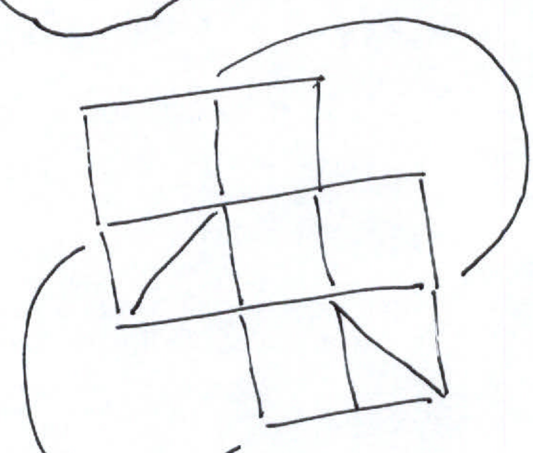


#5 Does the graph G_n (for $n = 1, 2, 3$) contain a Hamiltonian cycle? If not, why not? If so, find one.

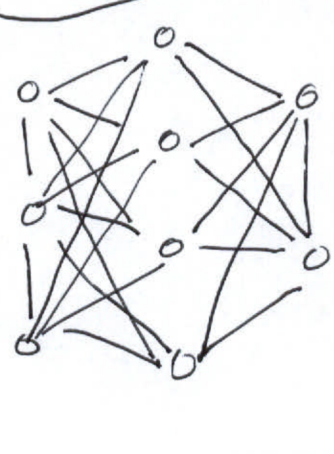
(a) G_1



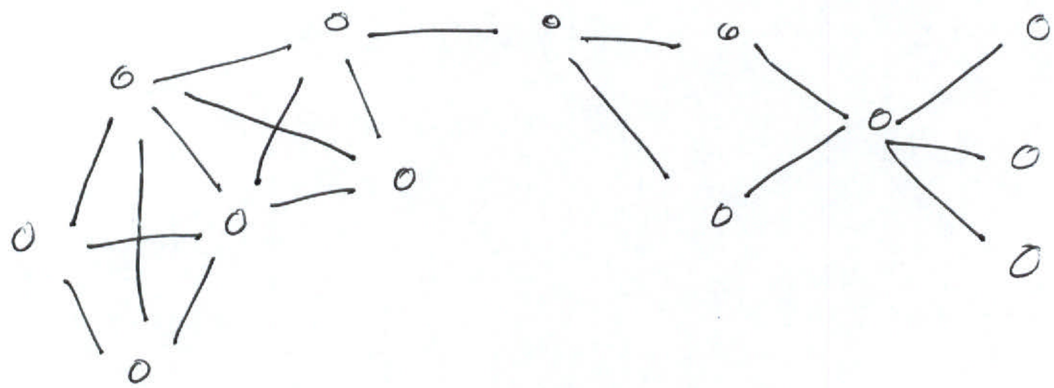
b) G_2



c) G_3



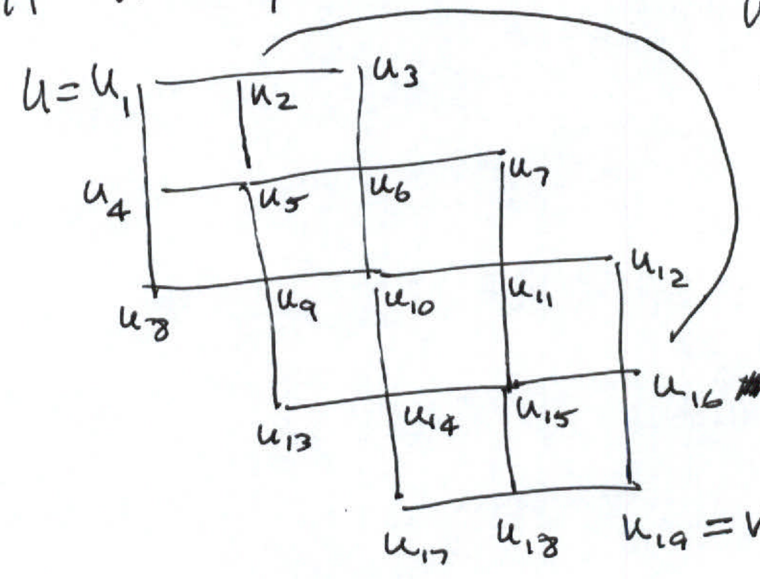
#6 Let G be the graph



Find $\alpha(G), \beta(G), \alpha_1(G), \beta_1(G)$.

Find a maximum matching in G.

#7 In the graph below find a minimal u-v separating set and a maximal set of internally disjoint u-v paths and also a minimal $u_{10}-u_{16}$ separating set and a maximal set of internally disjoint $u_{10}-u_{16}$ paths



#8 State whether each of the following is true or false. If true, give a proof. If false, give a counterexample.

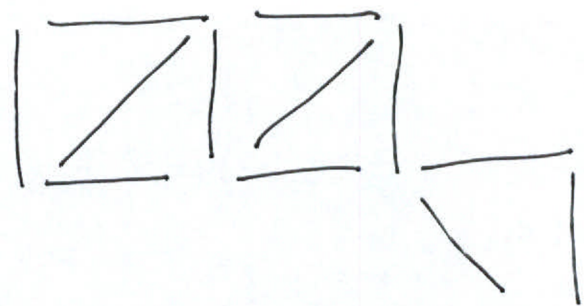
(a) If e is a bridge of G and v is incident to e, then v is a cut vertex.

(b) If v is a vertex of G and e is incident to v, then e is a bridge

(c) If S is a vertex cut in a connected graph G, then $G-S$ has exactly two components

(d) If X is an edge cut in a connected graph G, then $G-X$ has exactly two components

(e) If G is a connected graph with exactly two vertices of odd degree and $\deg v$ is odd, then at most one edge incident to v is a bridge.

#9 Let G = 

Find $C(G)$.

#10 Show that any nontrivial connected graph contains at least two vertices that are not cut vertices.

#11 Prove that a connected graph in which every vertex has degree 2 is a cycle.

#12 Let R be the relation on $E(G)$ where G is a nontrivial connected graph defined by eRf if $e, f \in E(G)$ and either $e = f$ or e and f lie on a common cycle.

(a) Prove R is an equivalence relation

(b) Show that if eRf then e and f belong to the same block of G.

#13 (a) For any integers $1 \leq m \leq n-1$ give an example of a connected graph with m cut vertices and n blocks.

(b) Prove that if G is a nontrivial connected graph with m cut vertices and n blocks then $m \leq n-1$

#14 You should know the definitions of:

cut-vertex
vertex cut, maximum vertex cut
maximal vertex cut
edge cut, maximum edge cut
maximal edge cut

$\kappa(G), \lambda(G), \delta(G)$
$\alpha(G), \alpha_1(G), \beta(G), \beta_1(G)$

u-v separating set
internally disjoint paths

blocks

matching
independent set of edges