Math 428 - Review problems for Final Exam - December ~~12~~ 14, 2009

This is the complete set of review problems.

\# 1 Let G be the complete bipartite graph with partite sets $\{a, b, c, d\}$ and $\{e, f, g\}$.
i) Find a bf-path of maximal length.
ii) Find a bc-path of maximal length.
iii) Find a trail of maximal length

\# 2 Let G and H be planar graphs.
a) Suppose $G + H$ is planar. Show that either G or H has order ≤ 2.
b) Does the converse of (a) hold. That is, if G and H are planar and G has order ≤ 2 does $G + H$ have to be planar?

\# 3 For each of the following sequences, answer the following questions (giving reasons).
i) Is the sequence graphical?
ii) If the sequence is the degree sequence of a graph G, can G be connected? Does G have to be connected.
iii) If the sequence is the degree sequence of a graph G, can G be Eulerian? Does G have to be Eulerian?
iv) If the sequence is the degree sequence of a graph G, can G be Hamiltonian? Does G have to be Hamiltonian?
v) If the sequence is the degree sequence of a graph G, can G be planar? Can G be maximal planar? Does G have to be planar or maximal planar?
a) $(5, 4, 4, 3, 3, 3)$
b) $(5, 4, 4, 4, 3, 3)$
c) $(5, 4, 4, 1, 1, 1)$
d) $(5, 5, 4, 4, 2, 2, 2, 2)$

\# 4 Let n be an integer, $n \geq 3$.
(a) How many isomorphism classes of $n-1$-regular graphs of order n are there? Why?
(b) How many isomorphism classes of $n-2$-regular graphs of order n are there? Why?

\# 5 Give an embedding of $C_3 \cup P_2$ as an induced subgraph of a 3-regular graph.

\# 6 Let v be a cut-vertex of a graph G. How many blocks of G can contain v?

\# 7 Prove that if B and C are blocks of a graph G then $|V(B) \cap V(C)| \leq 1$.

\# 8 Find all trees T such that $\overline{T}$ is planar. Explain.

\# 9 a) Show that $\gamma(K_5) = 1$.

b) Show that $\gamma(K_{4,3}) = 1$.

10 Suppose a graph G of order n has chromatic number n. Does G have to be complete? Why or why not?

11 Prove that a graph G of order ≥ 3 is connected if and only if G contains two distinct vertices u and v such that $G - u$ and $G - v$ are connected. (This is Theorem 1.10.)

12 Let G and$H$ be graphs with $V(G) \cap V(H) = \emptyset$. State the definitions of $G \cup H, G + H$, and $G \times H$.

13 Prove that two graphs G and H are isomorphic if and only if $\overline{G}$ and $\overline{H}$ are isomorphic.

14 Let F, G and H be graphs.

(a) Suppose that $F \cup H$ and $G \cup H$ are isomorphic. Do F and G have to be isomorphic. Explain your answer.

(b) Suppose that $F + H$ and $G + H$ are isomorphic. Do F and G have to be isomorphic. Explain your answer. Hint: Consider using problem #13.

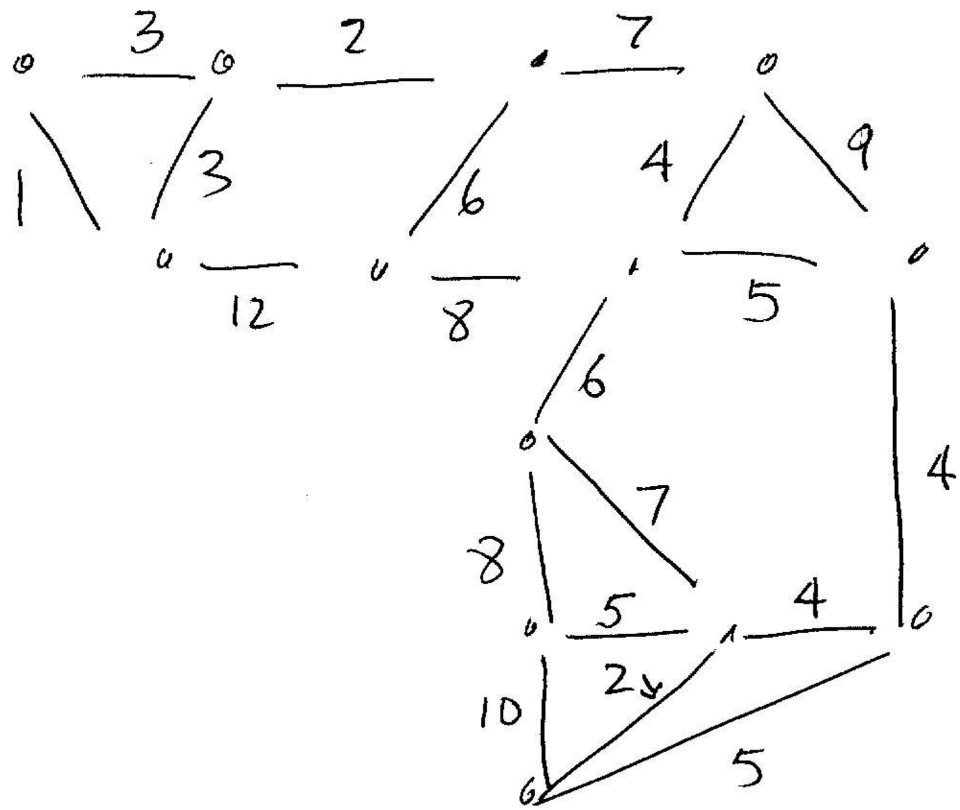
15(a) Suppose G has no cut vertices. How many bridges can G have? Explain.

(b) Suppose G has exactly one cut vertex. How many bridges can G have? Explain.

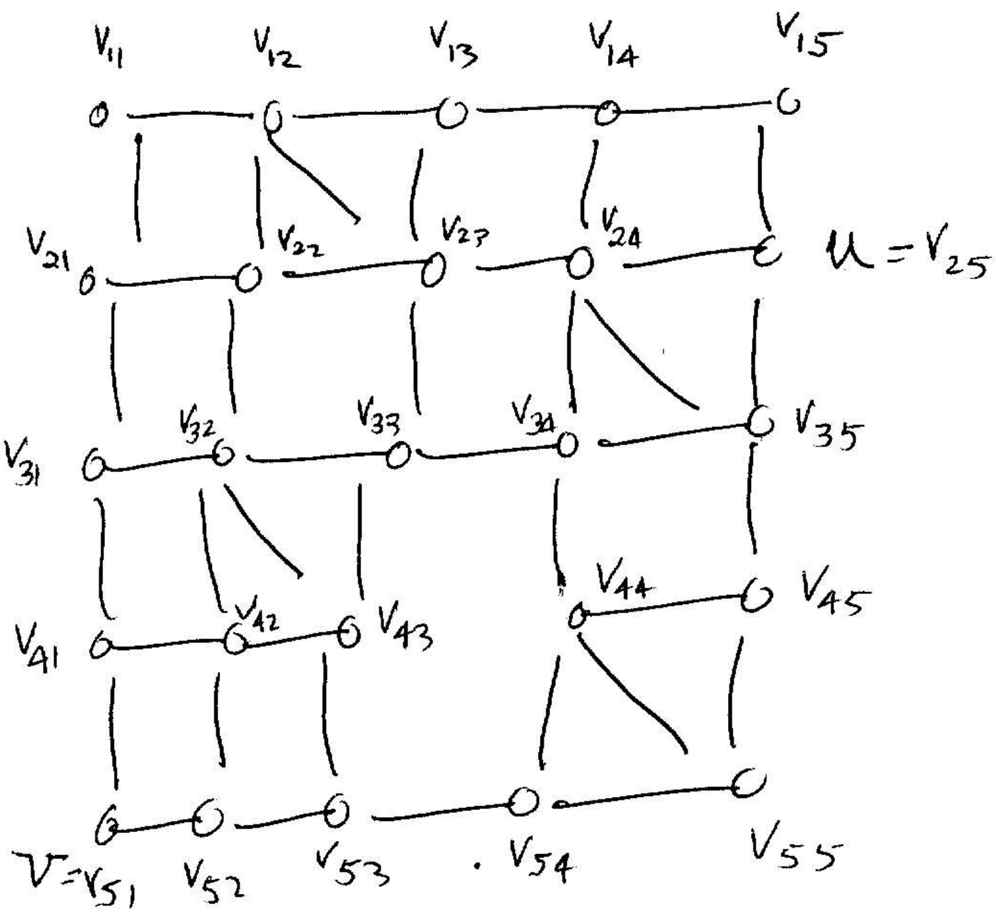
#16 Suppose that G is a graph of order n containing no isolate vertices. Prove that $\alpha(G) + \beta(G) = n$ and that $\alpha_1(G) + \beta_1(G) = n$.

#17 Let G be a graph. Prove that $\gamma(G) \leq 1 + \Delta(G)$ and that $\gamma(G) \leq 1 + max\{\delta(G)\}$ where the maximum is taken over all induced subgraphs.

#18 Find a minimal spanning tree in the following weighted graph using Kruskal's algorithm and also using Prim's algorithm. Show your work

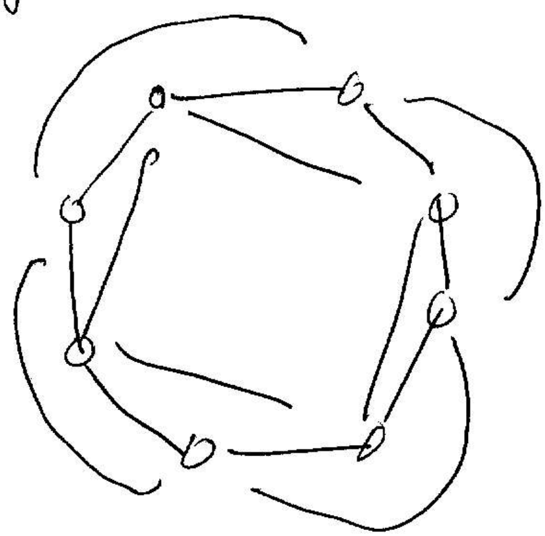


#19 Find a minimal u-v separating set and a maximal set of internally disjoint u-v paths in the following graph. Explain how Menger's theorem can be used to justify your answer

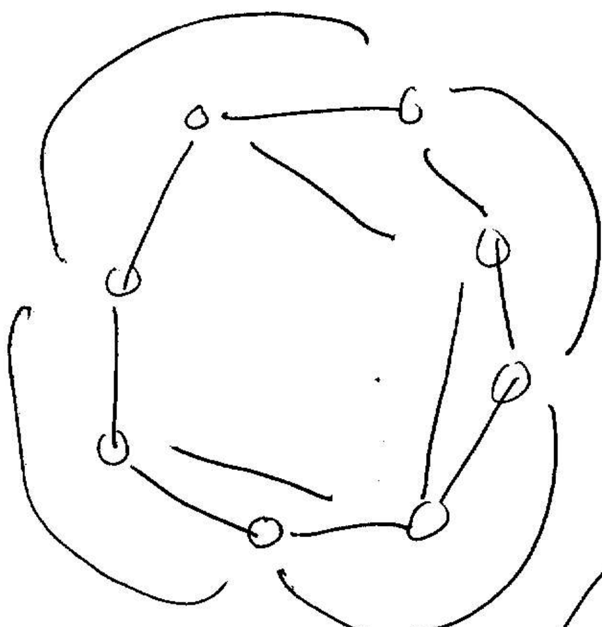


#20. In each of the following graphs find an ~~Euler~~ Eulerian circuit if on exists, or an Eulerian trail if one exist. If none exists explain why

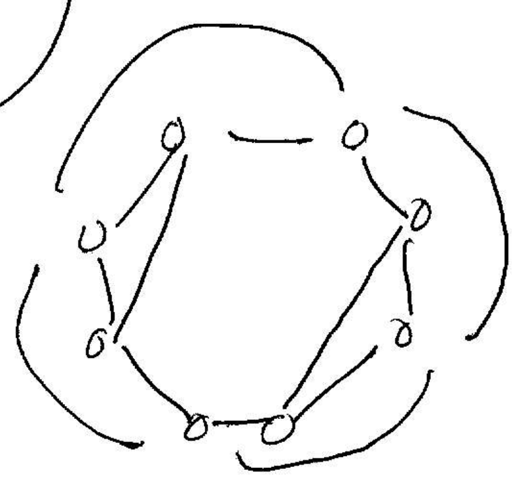
(a)



(b)



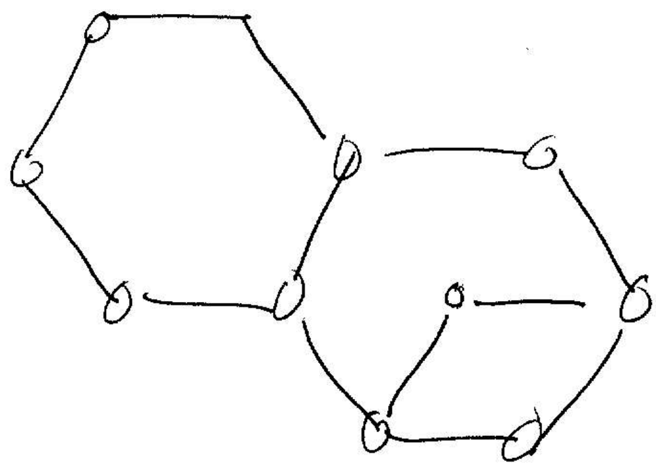
(c)



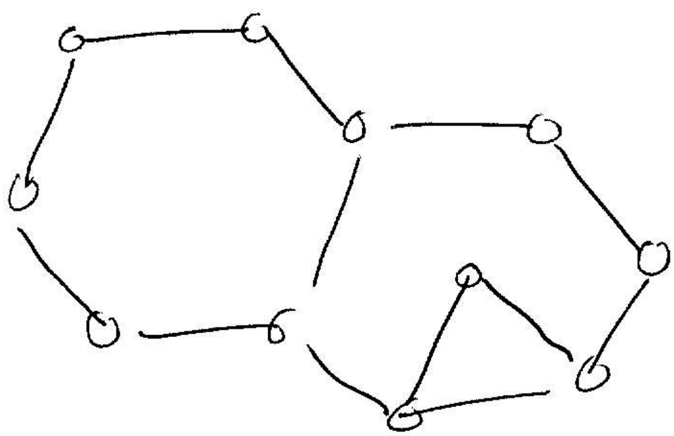
6

#21 Is each of the following graphs Hamiltonian. Why or why not

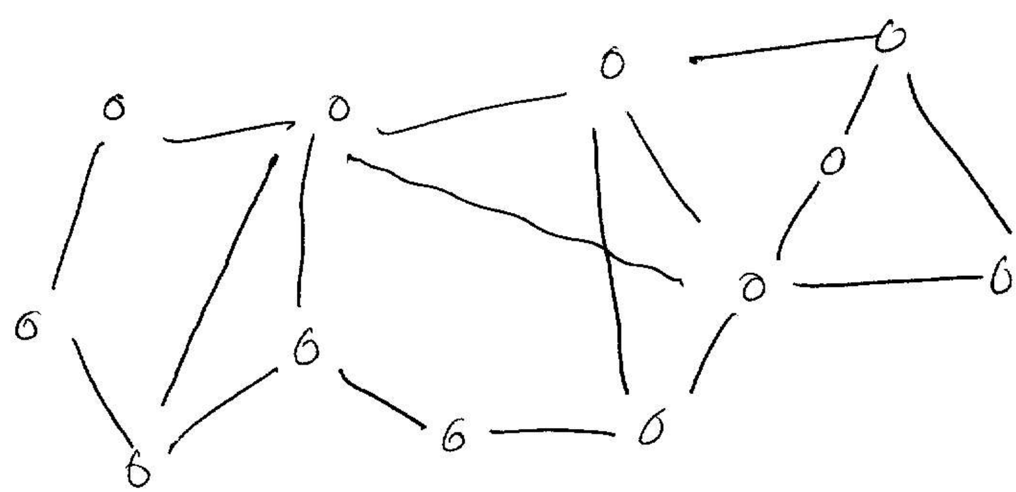
(a)



(b)

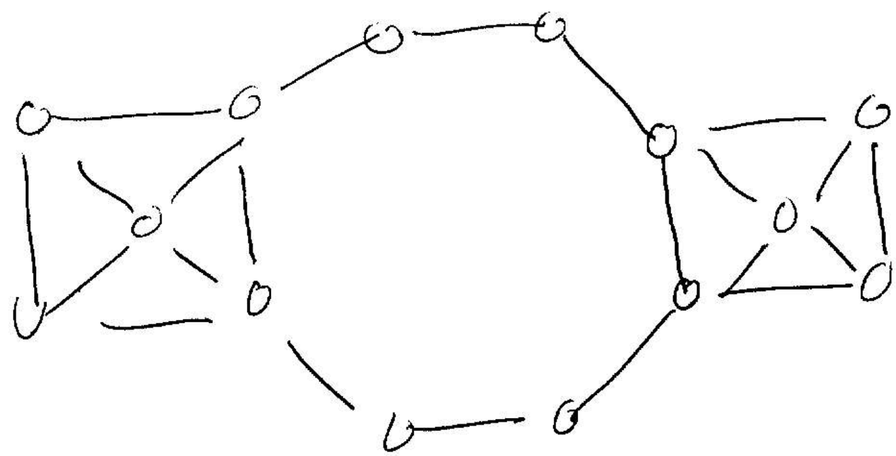


(c)



#22 Find the chromatic number of each of the following graphs

(a)



(b)