Math 477-02 - Review problems for Exam #2 - April 11, 2009
Review Session: Monday, April 13, 6:30-8:30 M in SEC-211 (BUSCH)

Calculators may not be used on the exam. You will be given a sheet containing a copy of table 5.1 of the the text and the following formulas:
Binomial: $P\{X = k\} = \binom{n}{k}p^k(1-p)^{n-k}, k = 0, 1, ..., n.E[X] = np, Var(X) = np(1-p)$.
Geometric: $P\{X = k\} = p(1-p)^{k-1}, k = 1, 2, ...E[x] = \frac{1}{p}, Var(X) = \frac{(1-p)}{p^2}$.
Poisson: $P\{X = k\} = \frac{\lambda^k}{k!}, k = 0, 1, 2, ...E[X] = \lambda, Var(X) = \lambda$.
Exponential: $f_X(x) = \lambda e^{-\lambda x}, x \geq 0, E[X] = \frac{1}{\lambda}, Var(X) = \frac{1}{\lambda^2}$.
Normal: $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}.E[x] = \mu, Var(X) = \sigma^2$.

Two additional problems (#9, #10) have been added on Monday, April 13.

#1 A continuous random variable X has density

$$f_X(x) = cx, \quad if \quad 0 \leq x \leq 1,$$

$$f_X(x) = 0, \quad if x < 0 \quad or \quad x > 1$$

for some constant c.
(a) Find c.
(b) Find $P\{X \geq \frac{1}{3}\}$ and $P\{X = \frac{1}{3}\}$.
(c) Find $E[X]$ and $Var(X)$.
Now assume that Y is a second continuous random variable which is uniforly distributed on the interval $[0, 1]$ an that X and Y are independent.
(d) Find the joint density function $f(x, y)$ being careful to specify where $f(x, y) = 0$ and giving its value where it is non-zero.
(e) Find $P\{X \geq Y\}$

#2 Ellen plays a game in which her chance of winning is $\frac{1}{5}$. Using a normal approximation, estimate the probability of her winning exactly 25 times if she plays the game 100 times.

#3 Alex, Bruce and Charlie are playing darts using the disk $x^2 + y^2 \leq 4$ as the target. They always hit the target, and the x and y components of their impact points have the following joint distributions ,denoted f_A, f_B, f_C respectively:

$$f_A(x, y) = c_A(4 - x^2 - y^2),$$

$$f_B(x, y) = c_B,$$

$$f_C(x,y) = c_C(x^2+y^2).$$

(a) Find c_A, c_B, c_C. (You might want to use polar coordinates for Alex and Charlie.)

(b) The game is scored by giving 4 points for a hit inside the circle $x^2+y^2=1$ and 1 point for a hit outside that circle. What is the expected value of the nuber of points scored by each player on a throw.

#4 A certain transistor has lifetime T, where T is a positive random variable, measured in days, with density$f(t) = Kte^{-2t}$.

(a) What is K?

(b) Suppose it is known that the component has lasted s days. What is the probability that it will last two more days?

#5 Let X be a binomial random variable with $n = 3$ and $p = .5$ and let Y be a geometric random variable with parameter $p = .5$. Suppose that X and Y are independent.

(a) Give the values $p(x,y)$ of the joint probability mass function of X and Y for all x, y satisfying $0 \leq x \leq 3, 1 \leq y \leq 4$.

(b) Find $P\{X < Y\}$.

(c) Find the probability mass function for $X + Y$.

#6 Cars pass a certain point on a road according to a Poisson process, with an average rate of 3 cars per hour. Find the probability that an observer wil see exacty three cars pass in one hour of observation

(a) if three cars pass in the first half hour;

(b) if no cars pass in the first half hour.

#7 By definition, a "hundred year flood" on a river is a flood which is so severe that it happens, on the average, once every hundred years. Find the probability that there will be exactly three hundred year floods on the Raritan River between 2010 and 2159 (inclusive), both exactly and by using a suitable Poisson approximation. Assume that at most one such flood can occur in any year and thet floods in different years are independent.

#8 Let X and Y be independent random variables, exponentially distributed with parameters λ and μ respectively.

(a) Find $P\{X > 2Y\}$.

(b) Find the probability density for the random variable $Z = X + Y$.

#9 A total of n balls, numbered $1, 2, ..., n$ are put into n urns, also numbered $1, 2, ..., n$ in such a way that ball number i is equally likely to go into any one of the urns numbered

$1, 2, ..., i$. Find:

(a) the expected number of urns that re empty;

(b) the probability that no urn is empty.

#10 An entomologist is catching mosquitos in certain region which is inhabited by r distinct types of mosquitos. Each mosquito caught will, independently of the types of the previous catches, be of type i with probability P_i (where, of course, $\sum_{1 \le i \le r} P_i = 1$).

(a) Compute the mean number of mosquitos that are caught before the first type 1 catch.

(b) Compute the mean number of types of insects that are caught before the first type 1 catch.