MATH 250-02 **February, 2012**

Practice questions for exam #1

#1 Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$,

$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$.

Find each of the following if it is defined. If it isn't defined, say so.

a) AC;
b) BC;
c) A^T;
d) $CA + 3A$;
e) $A\mathbf{v}$;
f) $2\mathbf{u} + 3\mathbf{v}$;
f) AC^T;
g) $CA - 2C$.

#2 (a) Is there a 5 by 7 matrix whose rows are linearly independent and whose columns are also linearly independent? Why or why not?

(b) Is there a 5 by 7 matrix whose rows are linearly independent and whose columns are linearly dependent? Why or why not?

(c) Is there a 5 by 7 matrix whose rows are linearly dependent and whose columns are linearly independent? Why or why not?

#3 a) Find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

b) Find all solutions of the system of equations

$$2x_1 + x_2 + 3x_4 = 5$$

$$x_3 = 0$$

$$x_1 - x_2 + 2x_4 = -1$$

$$x_2 + x_4 = 0.$$

#4 Let

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 & 3 \\ 2 & 1 & 1 & 1 & 1 \\ 4 & -1 & 1 & 3 & 5 \\ 1 & -4 & -1 & 2 & 2 \end{bmatrix}.$$

Apply Gaussian elimination to transform A to a matrix in reduced row echelon form (RREF).

#5 Let R be the reduced row echelon form of a matrix B.

(a) Is the span of the columns of B equal to the span of the columns of R? Explain why or why not.

(b) Is the span of the rows of B equal to the span of the rows of R? Explain why or why not.

#6 (a) Does the set of vectors $\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 0 \end{bmatrix} \}$ span $\mathbf{R}^3$? Why or why not?

(b) Does the set of vectors $\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 0 \\ 1 \end{bmatrix} \}$ span $\mathbf{R}^4$? Why or why not?

(c) Is the set of vectors in (a) linearly independent? Why or why not?

(d) Is the set of vectors in (b) linearly independent? Why or why not?

(e) Is the set of vectors $\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \\ 1 \end{bmatrix} \}$ linearly independent? Why or why not?

(f) Is the set of vectors $\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \}$ linearly independent? Why or why not?

#7 Consider the system of linear equations

$$x_1 + 3x_2 + 2x_3 + x_4 = 2$$

$$2x_1 - x_2 - 3x_3 + x_4 = 3$$

$$x_1 + x_2 + x_4 = 2$$

$$5x_2 + 5x_3 + x_4 = 1.$$

a) Write the augmented matrix corresponding to the system.
b) Find the general solution of this system and write it in vector form.

#8 You should be able to state the definitions of the following terms. (I give a page reference to the text for each term.)
the transpose of a matrix (page 7);
a linear combination of a set of vectors (page 14);
the standard vectors $\mathbf{e}_1, ..., \mathbf{e}_n$ (page 17);
a consistent (or inconsistent) system of linear equations (page 29);
the augmented matrix of a system of linear equations (page 31);
the coefficient matrix of a system of linear equations (page 31);
a basic variable for a system of linear equations (page 35);
a free variable for a system of linear equations (page 35);
the rank of a matrix (page 47);
the nullity of a matrix (page 47);
the phrase "a set of vectors is linearly dependent" (page 75);
the phrase "a set of vectors is linearly independent" (page 75);

#9 Let $A = \begin{bmatrix} 1 & -3 & 2 & 1 \\ 0 & -1 & 1 & 3 \\ 2 & 1 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -5 & 4 & 7 \\ 0 & -1 & 1 & 3 \\ 2 & 1 & -1 & 0 \end{bmatrix}$. Find an elementary matrix E such that $EA = B$ and an elementary matrix F such that $FB = A$.

#10 Find the LU decomposition of the matrix A in problem #9.

#11 Let A be a 2 by 4 matrix and R be its reduced row echelon form (RREF). Suppose the basic variables for $A\mathbf{x} = \mathbf{0}$ are x_1 and x_3 and that the general solution of $A\mathbf{x} = \mathbf{0}$ is

$$\mathbf{x} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

(a) Find R.
(b) Suppose

$$A = \begin{bmatrix} \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4 \end{bmatrix}$$

and

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find A.

#12 a) Find all real numbers a such that the vector $\begin{bmatrix} a \\ 2 \\ 1 \end{bmatrix}$ is in $Span\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\}$.

b) Find all real numbers c such that the system of equations

$$x_1 + x_3 = 1$$

$$2x_1 - x_2 = 1$$

$$x_1 + 3x_2 + 7x_3 = c$$

is consistent

#13 Give an example of:

a) a nonzero 2 by 2 matrix A which is not invertible;

b) a pair of 2 by 2 matrices B and C such that $BC \neq CB$.

#14 Let A be an n by n matrix in RREF. Prove that either $A = I_n$ or else the last row of A is zero.