MATH 250-02

03/28/2012

Practice questions for exam #2

#1 Compute the determinant of each of the following matrices:

a) $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & 0 & -1 & 1 \\ -1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ -1 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}$

#2 **Use Cramer's rule** to find x_3 if

$$2x_1 - x_2 = 3$$
$$-x_1 + 2x_2 - x_3 = 5$$
$$-x_2 + 2x_3 = 1.$$

Show your work.

#3 Find the characteristic polynomial, all real eigenvalues and bases for the corresponding eigenspaces for each of the following matrices:

a) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ -1 & 0 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix}$

#4 Let $A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

Find an invertible matrix P and a diagonal matrix D such that

$$D = P^{-1}AP.$$

Note that this is the same as requiring

$$A = PDP^{-1}.$$

#5 (a) Is the matrix

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

diagonalizable? Why or why not?

(b) Is the matrix

$$B = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 3 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

diagonalizable? Why or why not?

#6 You should be able to state the definitions of each of the following terms. (I give a page reference for each term.)

an upper triangular matrix (page 153);
a lower triangular matrix (page 153);
a subspace V of $\mathbf{R}^n$ (page 227);
the null space of a matrix A (page 232);
the column space of a matrix A (page 233);
the row space of a matrix (page 236);
a basis for a subspace V of $\mathbf{R}^n$ (page 241);
the dimension of a subspace V of $\mathbf{R}^n$ (page 246);
an eigenvector $\mathbf{v}$ of an n by n matrix A (page 294);
the eigenvalue λ of an n by n matrix A that corresponds to an eigenvector $\mathbf{v}$ (page 294);
the eigenspace of an n by n matrix A that corresponds to an eigenvalue λ (page 296);
the characteristic polynomial of an n by n matrix A (page 302);
the multiplicity of an eigenvalue λ of an n by n matrix A (page 305);
an n by n matrix A is diagonalizable if ... (page 315);

#7 Let $A = \begin{bmatrix} 0 & 3 & 0 & 2 & -3 \\ 1 & 2 & 2 & 3 & -2 \\ 2 & 4 & 4 & 5 & -4 \\ 1 & 1 & 2 & 1 & -1 \end{bmatrix}$.

(a) Find a basis for *Row A*
(b) Find a basis for *Col A*
(c) Find a basis for *Null A*

#8 Show that

$$\{\begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ -2 \\ 0 \\ 4 \end{bmatrix}\}$$

is a basis *Null A* where A is the matrix from problem #7.

#9 (a) Find the general solution of the system of differential equations:

$$y_1' = 2y_1 - y_2;$$

$$y_2' = -y_1 + 2y_2.$$

(Here y' denotes the derivative of y.)

(b) If

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

find A^{100}.

#10 Let A and B be 3 by 3 matrices. Suppose $det\ A = 2$ and $det\ B = 5$.
(a) Can you determine $det\ AB$? If so, what is it? If not, why not?
(b) Can you determine $det\ A^3$? If so, what is it? If not, why not?
(c) Can you determine $det\ 2B$? If so, what is it? If not, why not?

#11 Suppose A is an m by n matrix, that $rank\ A = 3$, $nullity\ A = 7$, and $nullity\ A^T = 5$. What are m and n?

#12 (a) Suppose that V and W are subspaces of $\mathbf{R}^n$ and that V is contained in W. Show that $dim\ V \leq dim\ W$.

(b) Suppose that V and W are subspaces of $\mathbf{R}^n$, that V is contained in W, and that $dim\ V = dim\ W$. Prove that $V = W$.

#13 Let A be an n by n matrix. Suppose that $\mathbf{u}$ is a 1-eigenvector for A, that $\mathbf{v}$ is a 2-eigenvector for A, and that $\mathbf{w}$ is a 3-eigenvector for A. Prove that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent.

#14 Let A be a 4 by 4 matrix. Suppose that $\mathbf{u}$ is a 1-eigenvector for A, that $\mathbf{v}$ is a 2-eigenvector for A, that $\mathbf{w}$ is a -3-eigenvector for A, and that $\mathbf{z}$ is a λ-eigenvector for A where λ is some negative number. Suppose that $rank\ [\mathbf{u}\ \mathbf{v}\ \mathbf{w}\ \mathbf{z}] = 3$. What is λ? Why?