**MATH 354-03** **March 31, 2005**

**Practice questions for exam #1**

I will work these problems at a review session on Sunday, 4/3, 3-5 PM in ARC-309 and again at a review session on Tuesday, 4/5, 7-9 PM in ARC-309. (ARC-309 is in the Mathematics and Science Learning Center on the third floor of ARC.) Remember that the exam is in class on Thursday, 4/7. Solutions to these problems will be posted after the 4/3 review session.

#1 Consider the linear programming problem

Maximize: $\mathbf{c}^T\mathbf{x}$
Subject to:
$A\mathbf{x} \leq \mathbf{b}$
$\mathbf{x} \geq \mathbf{0}$.

This problem is in standard form.

(a) State the dual problem.

(b) Show that if $\mathbf{x}$ is any feasible solution to the primal problem and $\mathbf{w}$ is any feasible solution to the dual problem then $\mathbf{c}^T\mathbf{x} \leq \mathbf{b}^T\mathbf{w}$.

(c) Show that if the primal problem is unbounded, then the dual problem is infeasible.

(d) Show that the dual of the dual of the given problem is the primal problem.

#2 Consider the linear programming problem

Maximize: $\mathbf{r}^T\mathbf{x}$
Subject to:
$A\mathbf{x} = \mathbf{s}$
$\mathbf{x} \geq \mathbf{0}$

This problem is in canonical form. Find the dual of this problem, by writing the primal problem in standard form and using your answer to #1. Explain why the dual involves unrestricted variables.

#3 Find the dual of the linear programming problem:

Minimize: $-3x_1 + 2x_2 + x_4$
Subject to:
$2x_1 + x_2 + x_3 + 2x_4 \geq 7$
$x_2 + 3x_4 = 5$
$x_1, x_2 \geq 0, x_3 \leq 0, x_4$ unrestricted.

#4 Use the revised simplex method to solve the linear programming problem

Maximize: $2x_1 + x_2 + 3x_3 + x_6 + 2x_7 + 3x_8$
Subject to:
$2x_1 + x_2 + x_4 + 3x_5 + x_7 \leq 24$
$x_1 + 3x_3 + x_4 + x_5 + 2x_6 + 3x_8 \leq 30$
$5x_1 + 3x_2 + 3x_4 + 2x_5 + x_7 + 5x_8 \leq 18$
$3x_1 + 2x_2 + x_3 + x_6 + 3x_8 \leq 20$
$x_1, ..., x_8 \geq 0.$

Give the current $B^{-1}$ and the current list of basic variables at each step.

#5 Consider the linear programming problem:

Maximize: $4x_1 + 3x_2 + 6x_3$
Subject to:
$3x_1 - 4x_2 - 6x_3 \leq 18$
$-2x_1 - x_2 + 2x_3 \leq 12$
$x_1 + 3x_2 + 2x_3 \leq 1$
$x_1, x_2, x_3 \geq 0.$

The optimal solution to this problem is $z = 4$ at the point $x_1 = 1, x_2 = 0, x_3 = 0$ and the final tableau for the simplex method is:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
|---|---|---|---|---|---|---|---|
| $x_4$ | 0 | $-13$ | $-12$ | 1 | 0 | $-3$ | 15 |
| $x_5$ | 0 | 5 | 6 | 0 | 1 | 2 | 14 |
| $x_1$ | 1 | 3 | 2 | 0 | 0 | 1 | 1 |
| | 0 | 9 | 2 | 0 | 0 | 4 | 4 |

(a) State the dual problem and find its optimal solution
(b) Find all values of $\Delta c_2$ such that the solution above remains optimal.
(c) Find all values of $\Delta c_5$ such that the solution above remains optimal.
(d) Find the optimal solution of the problem obtained by changing $c_6$ to 3.
(e) Suppose the final tableau is obtained from the initial tableau by multiplying by $B^{-1}$. Find $B^{-1}$.
(f) Find the optimal solution to the problem obtained by changing the constant term in the third constraint ($\mathbf{b}_3$) from 1 to 5.
(g) Find the optimal solution to the problem obtained by changing the constant term in the third constraint ($\mathbf{b}_3$) from 1 to 7.
(h) A further constraint $x_2 + x_3 \geq 1$ is added to the original problem. Use the dual simplex method to find an optimal solution to this new problem (if one exists).
(i) A different further constraint $2x_1 + x_2 \leq 1$ is added to the original problem (not to the modified problem in (h)). Use the dual simplex method to find an optimal solution to this new problem (if one exists).

#6 Find an optimal solution to the following pure integer programming problem.

Maximize: $x_1 + 3x_2$
Subject to:
$x_1 - 2x_2 \geq 0$
$x_1 + 2x_2 \leq 42$
$x_1, x_2 \geq 0,\ x_1, x_2$ integers

#7 Find an optimal solution to the following pure integer programming problem.

Maximize: $x_1 + 2x_2 + x_3 + x_4$
Subject to:

$2x_1 + x_2 + 3x_3 + x_4 \leq 8$
$2x_1 + 3x_2 + 4x_4 \leq 12$
$3x_1 + x_2 + 2x_3 \leq 18$
$x_1, x_2, x_3, x_4 \geq 0,\ x_1, x_2, x_3, x_4$ integers