Math 135, Section C7 - Review problems for Exam #1 - June 14, 2010

#1 Find all x such that $|x-3| < \frac{7}{2}$ and express your answer in interval notation.

#2 Write an equation for a staight line:
(a) which passes throught the point $(1, -2)$ and has slope 3;
(b) which passes through the points $(3, 5)$ and $(5, -8)$
(c) which passes through the point $(-4, 1)$ and is parallel to the the staight line with equation y = -2x + 7;
(d) which passes through the point $(-1, 1)$ and is perpendicular to the line through $(7, 4)$ and $(2, 2)$.

#3 Write an equation of the circle with center $(3, 2)$ and radius 5.

#4 (a) The graph of the equation $x^2 + y^2 - 2x + 4y - 4 = 0$ is a circle. What are the center and raduis of this circle?
(b) The graph of the equation $7x - 5y + 23 = 0$ is a straight line. What is its slope? If the point $(a, 2a)$ is on this line, what is a?

#5 Suppose that $f(x) = 2x - 1$ if $x < 1, f(1) = a$ and $f(x) = 3x + b$ if $x > 1$. Suppose further that $f(x)$ is continuous at $x = 1$. What are a and b? Explain why using the definition of continuity.

#6 Find each of the following limits or state that the limit does not exist:

(a) $lim_{x\to 2}(x^2 + \frac{x}{x-1})$

(b) $lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$

(c) $lim_{x\to 2^+}\frac{x-2}{|x-2|}$

(d) $lim_{x\to 2^-}\frac{x-2}{|x-2|}$

(e) $lim_{x\to 2}\frac{x-2}{|x-2|}$

(f) $lim_{x\to 2^+}\frac{1}{|x-2|}$

(g) $lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}$

#7 Use the definition of derivative to find

(a) $f'(x)$ if $f(x) = x^2 + x + 1$

(b) $g'(x)$ if $g(x) = \frac{2}{x+1}$

(c) $h'(x)$ if $h(x) = \sqrt{2x+3}$

#8 In each part, find $f'(x)$ by any method:

(a) $f(x) = x^3 + 2x^2 - x + 3$

(b) $f(x) = x\sqrt{x} + 3\frac{1}{x\sqrt{x}}$

(c) $f(x) = sin(2x+3)$

(d) $e^{(2x+3)}$

(e) $f(x) = e^{sin(x)}$

(f) $f(x) = \frac{sin(x)}{e^{2x+3}}$

(g) $f(x) = \sqrt{\frac{x^2+1}{x^2+2}}$

(h) $f(x) = (x^3 + 2x)^{17}$

(i) $f(x) = \ln(\sin(2x+3))$

(j) $f(x) = x^2 \sin(e^{2x} + 3)$

#9 A straight east-west road goes through the town of Bend. Suppose that at time t (in hours), where $0 \leq t \leq 10$, a car is $20 + 8t - t^2$ miles east of Bend.

(a) What it the velocity of the car at time t?

(b) What is the speed of the car at time t?

(c) What is the acceleration of the car at time t?

(d) What it is the total distance traveled by the car between $t = 1$ and $t = 7$?

#10 Suppose $f(x)$ and $g(x)$ are two functions which are defined for all real numbers. Suppose that
$f(-2) = 1, f(-1) = 0, f(0) = 2, f(1) = 1, f(2) = -1,$
$g(-2) = -2, g(-1) = 1, g(0) = 0, g(1) = -2, g(2) = 2,$
$f'(-2) = 0, f'(-1) = 3, f'(0) = -3, f'(1) = 2, f'(2) = -1,$
$g'(-2) = 2, g'(-1) = -1, g'(0) = 2, g'(1) = -2,$ and $g'(2) = 3.$
Let $h(x) = f(g(x))$ and $p(x) = g(f(x))$. Find:
(a) h(2)

(b) h'(2)
(c) p(2)
(d) p'(2)

#11 Let

$$f(x) = 1 - x^2, \text{if } x < 2;$$

and

$$f(x) = ax + b, \text{if } x \geq 2.$$

Suppose $f(x)$ is differentiable at $x = 2$. What are a and b? Why?

#12
Let

$$g(x) = x + 3, \text{if } x \leq 1;$$

and

$$g(x) = x^2 + 3, \text{if } x > 1.$$

Is $g(x)$ continuous at $x = 1$? Is $g(x)$ differentiable at $x = 1$? Explain your answers using the definitions.

#13 Show that

$$t^2 + 1 = \frac{10}{3t^2 + 2}$$

for some t in the interval $[-2, 2]$.

#14 A block of ice in the shape of a cube originally has volume $1,000$ cubic centimeters. It is melting in such a way that it maintains its cubical shape at all times and that the length of each of its edges is decreasing at the rate of 1 centimeter per hour. At what rate is its surface area decreasing at the time its volume is 27 cubic centimeters?

#15 Find an equation for the tangent line to the graph of $y = e^{x^2+3}$ at the point where $x = 1$.