CONTEMPORARY MATHEMATICS

557

Representation Theory and Mathematical Physics

Conference in Honor of
Gregg Zuckerman's 60th Birthday
October 24–27, 2009
Yale University

Jeffrey Adams
Bong Lian
Siddhartha Sahi
Editors

American Mathematical Society

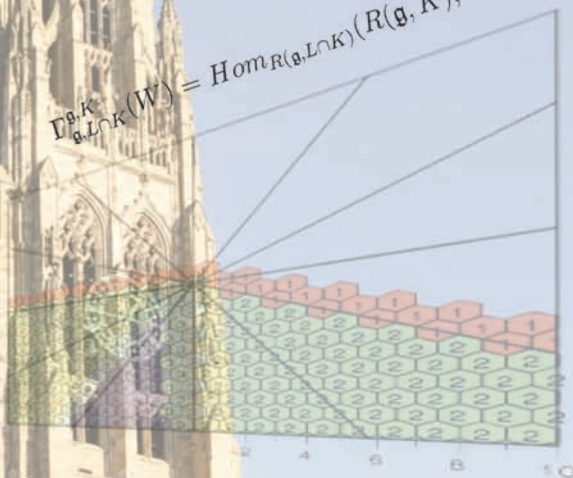
http://dx.doi.org/10.1090/conm/557

Representation Theory and Mathematical Physics

Representation Theory and Mathematical Physics
Conference in honor of Gregg Zuckerman's 60^{th} Birthday

Yale University, October 24-27, 2009

$$\Gamma^{\mathfrak{g},K}_{\mathfrak{g},L\cap K}(W) = Hom_{R(\mathfrak{g},L\cap K)}(R(\mathfrak{g},K),W)_K$$



$$\{u,v\} = (-1)^{|u|} Res_w Res_{z-w}[b(z-w)u](w)v$$

Speakers:

Barbasch, Dan (Cornell)
Frenkel, Igor (Yale)
Gaitsgory, Dennis (Harvard)
Garland, Howard (Yale)
Howe, Roger (Yale)
Kostant, Bert (MIT)
Kobayashi, Toshiyuki (Tokyo)
Lusztig, George (MIT)
Penkov, Ivan (Jacobs University, Bremen)
Sally, Paul (Chicago)
Schmid, Wilfried (Harvard)
Serganova, Vera (Berkeley)
Speh, Birgit (Cornell)
Stein, Elias (Princeton)
Trapa, Peter (University of Utah)
Vogan, David (MIT)
Willenbring, Jeb (University of Wisconsin, Milwaukee)

Organizers:

Adams, Jeffrey (University of Maryland)
Kapranov, Mikhail (Yale)
Lian, Bong (Brandeis)
Sahi, Siddhartha (Rutgers)

Sponsors:

National Science Foundation
Yale University

Contact:

Website: http://www.liegroups.org/zuckerman/
Local contact: Mary Ellen DelVecchio, *mary.delvecchio@yale.edu* 203-432-7058
Poster background: courtesy of http://media4.its.yale.edu/res/sites/OPA/

CONTEMPORARY MATHEMATICS

557

Representation Theory and Mathematical Physics

Conference in Honor of
Gregg Zuckerman's 60th Birthday
October 24–27, 2009
Yale University

Jeffrey Adams
Bong Lian
Siddhartha Sahi
Editors

American Mathematical Society
Providence, Rhode Island

2010 *Mathematics Subject Classification.* Primary 22E45, 22E46, 22E47, 17B65, 17B68, 17B69, 33D52.

Library of Congress Cataloging-in-Publication Data

Representation theory and mathematical physics : conference in honor of Gregg Zuckerman's 60th birthday, October 24–27, 2009, Yale University / Jeffrey Adams, Bong H. Lian, Siddhartha Sahi, editors.

p. cm. — (Contemporary mathematics ; v. 557)

Includes bibliographical references.

ISBN 978-0-8218-5246-0 (alk. paper)

1. Linear algebraic groups—Congresses. 2. Representations of Lie groups—Congresses. 3. Mathematical physics—Congresses. I. Zuckerman, Gregg. II. Adams, Jeffrey. III. Lian, Bong H., 1962- IV. Sahi, Siddhartha, 1958-

QA179.R47 2011
515′.7223—dc23

2011030808

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1   16 15 14 13 12 11

Dedicated to Gregg Zuckerman on the occasion of his 60^{th} birthday.

Contents

Preface

Lie groups and their representations are a fundamental area of mathematics, with connections to geometry, topology, number theory, physics, combinatorics, and many other areas. Gregg Zuckerman's work lies at the very heart of the modern theory of representations of Lie groups. His influential ideas on derived functors, the translation principle, and coherent continuation laid the groundwork of modern algebraic theory.

Zuckerman has long been active in the fruitful interplay between mathematics and physics. Developments in this area include work on chiral algebras, and the representation theory of affine Kac-Moody algebras. Recent progress on the geometric Langlands program points to exciting connections between automorphic representations and dual fibrations in geometric mirror symmetry.

These topics were the subject of a conference in honor of Gregg Zuckerman's 60th birthday, held at Yale, October 24-27, 2009.

Summary of Contributions

The classical Plancherel theorem is a statement about the Fourier transform on $L^2(\mathbb{R})$. It has generalizations to any locally compact group. *The Plancherel Formula, The Plancherel Theorem, and the Fourier Transform of Orbital Integrals* by Rebecca A. Herb and Paul J. Sally, Jr. surveys the history of this subject for non-abelian Lie groups and p-adic groups.

One of Zuckerman's major contributions to representation theory is the technique now known as cohomological induction or the derived functor construction of representations. An important special case of this construction are the so-called $A_q(\lambda)$ representations which are cohomologically induced from one-dimensional characters. The paper *Branching Problems of Zuckerman Derived Functor Modules* by Toshiyuki Kobayashi provides a comprehensive survey of known results on the restrictions of the $A_q(\lambda)$ to symmetric subgroups, along with sketches of the most important ideas of the proofs.

Chiral Equivariant Cohomology of Spheres, by Bong H. Lian, Andrew R. Linshaw, and Bailin Song, is a survey of their work on the theory of chiral equivariant cohomology. This is a new topological invariant which is vertex algebra valued and contains the Borel-Cartan equivariant cohomology theory of a G-manifold as a substructure. The paper describes some of the general structural features of the new invariant—a quasi-conformal structure, equivariant homotopy invariance, and the values of this cohomology on homogeneous spaces—as well as a class of group actions on spheres having the same classical equivariant cohomology, but which can all be distinguished by the new invariant.

An irreducible admissible representation of a Lie group G is determined by its global character, which is an invariant distribution. It is represented by a conjugation invariant function defined on a dense subset of the semisimple elements. In *Computing Global Characters* Jeffrey Adams gives an explicit algorithm for such characters, based on the Kazhdan-Lusztig-Vogan polynomials.

Part of Arthur's conjectures predict the existence of certain stable virtual characters associated to nilpotent orbits. These conjectures are known in the case of an Archimedean field. In *Stable Combinations of Special Unipotent Representations*, Dan M. Barbasch and Peter E. Trapa study the space of stable virtual representations associated certain nilpotent orbits $\mathcal{O}$, and relate this space to the *special piece* of $\mathcal{O}$.

Levi Components of Parabolic Subalgebras of Finitary Lie Algebras, by Elizabeth Dan-Cohen and Ivan Penkov, turns to understanding certain "standard" subalgebras of the finitary Lie algebras including $sl(\infty)$, $so(\infty)$ and $sp(\infty)$. A lot is known about their Cartan, Borel and parabolic subalgebras. This paper goes much further by giving a description of subalgebras which can appear as the Levi component of a simple finitary Lie algebra, and a characterization of all parabolic subalgebras of which a given subalgebra is a Levi component.

In *On Extending the Langlands-Shahidi Method to Arithmetic Quotients of Loop Groups*, Howard Garland describes an important new approach to proving analytic properties of L-functions on certain finite dimensional groups by using the theory of Eisenstein series on infinite dimensional loop groups. This theory of Eisenstein series on loop groups for minimal parabolics has been developed by him in a series of previous papers. In this paper, he shows how the theory changes when one works with Eisenstein series associated to cusp forms on maximal parabolics.

If a group K acts on a vector space V then one is often interested in understanding the algebra of invariant polynomials $S\left(V^*\right)^K$ as explicitly as possible. In the paper *The Measurement of Quantum Entanglement and Enumeration of Graph Coverings,* motivated by questions in quantum computing, Michael W. Hero, Jeb F. Willenbring, and Lauren Kelly Williams consider the case of the group $K = U(n_1) \times \cdots \times U\left(n_r\right)$ acting in the usual manner on the tensor product $V = \mathbb{C}^{n_1} \otimes \cdots \otimes \mathbb{C}^{n_r}$, and they determine the invariant polynomials in the limiting case where all the $n_i \to \infty$.

Roger Howe's theory of dual pairs typically refers to a commuting pair of subgroups of the symplectic group. Although Howe's original formulation was in terms of Lie superalgebras, and such algebras have a wide variety of applications, their dual pairs have not received much attention. In *The Dual Pairs* $(O_{(p,q)}, \widetilde{OSp}_{2,2})$ *and Zuckerman Translation*, Dan Lu and Roger Howe study the representation theory of the dual pair consisting of the group $O(p,q)$ and the Lie superalgebra $\widetilde{OSp}_{2,2}$.

The paper *On the algebraic set of singular elements in a complex simple Lie algebra* by Bertram Kostant and Nolan Wallach studies a space of defining equations, denoted M, for the cone of singular elements of a semisimple Lie algebra $\mathfrak{g}$. The main result of the paper is the explicit decomposition of M as a $\mathfrak{g}$-module.

In *An Explicit Embedding of Gravity and the Standard Model in* E_8, Garrett Lisi gives some preliminary steps in the construction of a unified theory of all interactions based on the gauge group E_8. The kinematic framework he proposes accounts for one of the three observed generations of matter, but also include many particles that are not yet observed. His description includes an explicit embedding of the standard model and gravitational gauge groups into E_8, and the action of the corresponding Lie algebra generators on fermions.

Harmonic analysis on a connected reductive algebraic group G is a special case of harmonic analysis on symmetric spaces: one writes $G = G \times G/G^\delta$ where G^δ is the fixed points of the involution interchanging the two factors. In *From Groups to Symmetric Spaces* George Lusztig studies various properties which are known in the group case, and to what extent they generalize to other symmetric spaces.

In *Study of Antiorbital Complexes* George Lusztig studies a problem on the support of the Fourier transform over a finite field. Special cases are related to cuspidal character sheaves and canonical bases.

A "classical" automorphic representation V of a group G corresponds to an imbedding $V \hookrightarrow L^2\,(G/\Gamma)$. Composing this with evaluation at identity, one obtains an automorphic distribution. The paper *Adelization of Automorphic Distributions and Mirabolic Eisenstein Series* by Stephen D. Miller and Wilfried Schmid analyzes the mirabolic Eisenstein series attached to a congruence subgroup of $GL(n, Z)$ and its associated automorphic distribution.

In an algebraic setting, Ivan Penkov and Vera Serganova propose a systematic approach to studying various categories of modules over the Lie algebras $sl(\infty)$, $o(\infty)$ and $sp(\infty)$. One of the main results of *Categories of Integrable* $sl(\infty)$*-,* $o(\infty)$*-,* $sp(\infty)$*-modules* states that an integrable module with finite dimensional weight spaces is semisimple. Another important result gives a description of the largest category of integrable modules which is closed under dualization and whose objects have finite Loewy lengths.

Macdonald polynomials are a far-reaching generalization of a number of important special functions in representation theory and combinatorics. The expansion coefficients of a product of two Macdonald polynomials may be regarded as generalized Littlewood-Richardson coefficients. The paper *Binomial Coefficients and Littlewood–Richardson Coefficients for Interpolation Polynomials and Macdonald Polynomials* by Siddhartha Sahi obtains explicit formulas for these coefficients by solving a more general problem involving the interpolation polynomials introduced by Knop and Sahi.

The paper *Restriction of some Representations of* $U(p, q)$ *to a Symmetric Subgroup* by Birgit Speh studies the restriction of derived functor modules of $U(p, q)$ to certain noncompact symmetric subgroups. It is shown that in various cases the decomposition is discrete with finite multiplicities.

Titles in This Series

557 **Jeffrey Adams, Bong Lian, and Siddhartha Sahi, Editors,** Representation theory and mathematical physics, 2011

556 **Leonid Gurvits, Philippe Pébay, J. Maurice Rojas, and David Thompson, Editors,** Randomization, relaxation, and complexity in polynomial equation solving, 2011

555 **Alberto Corso and Claudia Polini, Editors,** Commutative algebra and its connections to geometry, 2011

554 **Mark Agranovsky, Matania Ben-Artzi, Greg Galloway, Lavi Karp, Simeon Reich, David Shoikhet, Gilbert Weinstein, and Lawrence Zalcman, Editors,** Complex analysis and dynamical systems IV. Part 2. General relativity, geometry, and PDE, 2011

553 **Mark Agranovsky, Matania Ben-Artzi, Greg Galloway, Lavi Karp, Simeon Reich, David Shoikhet, Gilbert Weinstein, and Lawrence Zalcman, Editors,** Complex analysis and dynamical systems IV. Part 1. Function theory and optimization, 2011

551 **Jesus Araujo-Gomez, Bertin Diarra, and Alain Escassut, Editors,** Advances in non-Archimedean analysis, 2011

552 **Robert Sims and Daniel Ueltschi, Editors,** Entropy and the quantum II, 2011

550 **Y. Barkatou, S. Berhanu, A. Meziani, R. Meziani, and N. Mir, Editors,** Geometric analysis of several complex variables and related topics, 2011

549 **David Blázquez-Sanz, Juan J. Morales-Ruiz, and Jesús Rodríguez Lombardero, Editors,** Symmetries and related topics in differential and difference equations, 2011

548 **Habib Ammari, Josselin Garnier, Hyeonbae Kang, and Knut Sølna, Editors,** Mathematical and statistical methods for imaging, 2011

547 **Krzysztof Jarosz, Editor,** Function spaces in modern analysis, 2011

546 **Alain Connes, Alexander Gorokhovsky, Matthias Lesch, Markus Pflaum, and Bahram Rangipour, Editors,** Noncommutative geometry and global analysis, 2011

545 **Christian Houdré, Michel Ledoux, Emanuel Milman, and Mario Milman, Editors,** Concentration, functional inequalities and isoperimetry, 2011

544 **Carina Boyallian, Esther Galina, and Linda Saal, Editors,** New Developments in Lie Theory and its Applications, 2011

543 **Robert S. Doran, Paul J. Sally, Jr., and Loren Spice, Editors,** Harmonic analysis on reductive, p-adic groups, 2011

542 **E. Loubeau and S. Montaldo, Editors,** Harmonic maps and differential geometry, 2011

541 **Abhijit Champanerkar, Oliver Dasbach, Efstratia Kalfagianni, Ilya Kofman, Walter Neumann, and Neal Stoltzfus, Editors,** Interactions between hyperbolic geometry, quantum topology and number theory, 2011

540 **Denis Bonheure, Mabel Cuesta, Enrique J. Lami Dozo, Peter Takáč, Jean Van Schaftingen, and Michel Willem, Editors,** Nonlinear Elliptic Partial Differential Equations, 2011

539 **Kurusch Ebrahimi-Fard, Matilde Marcolli, and Walter D. van Suijlekom, Editors,** Combinatorics and physics, 2011

538 **José Ignacio Cogolludo-Agustín and Eriko Hironaka, Editors,** Topology of algebraic varieties and singularities, 2011

537 **César Polcino Milies, Editor,** Groups, algebras and applications, 2011

536 **Kazem Mahdavi, Debbie Koslover, and Leonard L. Brown III, Editors,** Cross disciplinary advances in quantum computing, 2011

535 **Maxim Braverman, Leonid Friedlander, Thomas Kappeler, Peter Kuchment, Peter Topalov, and Jonathan Weitsman, Editors,** Spectral theory and geometric analysis, 2011

534 **Pere Ara, Fernando Lledó, and Francesc Perera, Editors,** Aspects of operator algebras and applications, 2011

533 **L. Babinkostova, A. E. Caicedo, S. Geschke, and M. Scheepers, Editors,** Set theory and its applications, 2011

532 **Sergiy Kolyada, Yuri Manin, Martin Möller, Pieter Moree, and Thomas Ward, Editors,** Dynamical numbers: Interplay between dynamical systems and number theory, 2010

531 **Richard A. Brualdi, Samad Hedayat, Hadi Kharaghani, Gholamreza B. Khosrovshahi, and Shahriar Shahriari, Editors,** Combinatorics and graphs, 2010

530 **Vitaly Bergelson, Andreas Blass, Mauro Di Nasso, and Renling Jin, Editors,** Ultrafilters across Mathematics, 2010

529 **Robert Sims and Daniel Ueltschi, Editors,** Entropy and the Quantum, 2010

528 **Alberto Farina and Enrico Valdinoci, Editors,** Symmetry for Elliptic PDEs, 2010

527 **Ricardo Castaño-Bernard, Yan Soibelman, and Ilia Zharkov, Editors,** Mirror symmetry and tropical geometry, 2010

526 **Helge Holden and Kenneth H. Karlsen, Editors,** Nonlinear partial differential equations and hyperbolic wave phenomena, 2010

525 **Manuel D. Contreras and Santiago Díaz-Madrigal, Editors,** Five lectures in complex analysis, 2010

524 **Mark L. Lewis, Gabriel Navarro, Donald S. Passman, and Thomas R. Wolf, Editors,** Character theory of finite groups, 2010

523 **Aiden A. Bruen and David L. Wehlau, Editors,** Error-correcting codes, finite geometries and cryptography, 2010

522 **Oscar García-Prada, Peter E. Newstead, Luis Álverez-Cónsul, Indranil Biswas, Steven B. Bradlow, and Tomás L. Gómez, Editors,** Vector bundles and complex geometry, 2010

521 **David Kohel and Robert Rolland, Editors,** Arithmetic, geometry, cryptography and coding theory 2009, 2010

520 **Manuel E. Lladser, Robert S. Maier, Marni Mishna, and Andrew Rechnitzer, Editors,** Algorithmic probability and combinatorics, 2010

519 **Yves Félix, Gregory Lupton, and Samuel B. Smith, Editors,** Homotopy theory of function spaces and related topics, 2010

518 **Gary McGuire, Gary L. Mullen, Daniel Panario, and Igor E. Shparlinski, Editors,** Finite fields: Theory and applications, 2010

517 **Tewodros Amdeberhan, Luis A. Medina, and Victor H. Moll, Editors,** Gems in experimental mathematics, 2010

516 **Marlos A.G. Viana and Henry P. Wynn, Editors,** Algebraic methods in statistics and probability II, 2010

515 **Santiago Carrillo Menéndez and José Luis Fernández Pérez, Editors,** Mathematics in finance, 2010

514 **Arie Leizarowitz, Boris S. Mordukhovich, Itai Shafrir, and Alexander J. Zaslavski, Editors,** Nonlinear analysis and optimization II, 2010

513 **Arie Leizarowitz, Boris S. Mordukhovich, Itai Shafrir, and Alexander J. Zaslavski, Editors,** Nonlinear analysis and optimization I, 2010

512 **Albert Fathi, Yong-Geun Oh, and Claude Viterbo, Editors,** Symplectic topology and measure preserving dynamical systems, 2010

511 **Luise-Charlotte Kappe, Arturo Magidin, and Robert Fitzgerald Morse, Editors,** Computational group theory and the theory of groups, II, 2010

510 **Mario Bonk, Jane Gilman, Howard Masur, Yair Minsky, and Michael Wolf, Editors,** In the Tradition of Ahlfors-Bers, V, 2010

509 **Primitivo B. Acosta-Humánez and Francisco Marcellán, Editors,** Differential algebra, complex analysis and orthogonal polynomials, 2010

For a complete list of titles in this series, visit the
AMS Bookstore at **www.ams.org/bookstore/**.

This volume contains the proceedings of the conference on Representation Theory and Mathematical Physics, in honor of Gregg Zuckerman's 60th birthday, held October 24–27, 2009, at Yale University.

Lie groups and their representations play a fundamental role in mathematics, in particular because of connections to geometry, topology, number theory, physics, combinatorics, and many other areas. Representation theory is one of the cornerstones of the Langlands program in number theory, dating to the 1970s. Zuckerman's work on derived functors, the translation principle, and coherent continuation lie at the heart of the modern theory of representations of Lie groups. One of the major unsolved problems in representation theory is that of the unitary dual. The fact that there is, in principle, a finite algorithm for computing the unitary dual relies heavily on Zuckerman's work.

In recent years there has been a fruitful interplay between mathematics and physics, in geometric representation theory, string theory, and other areas. New developments on chiral algebras, representation theory of affine Kac-Moody algebras, and the geometric Langlands correspondence are some of the focal points of this volume. Recent developments in the geometric Langlands program point to exciting connections between certain automorphic representations and dual fibrations in geometric mirror symmetry.

ISBN 978-0-8218-5246-0

9 780821 852460

CONM/557