EXAM I, FALL 2014

1. a. Solve $|x-3|=7$.

x =	$10, -4$

b. Find the center and radius of the circle described by $x^2+y^2-2x+4y=0$.

center =	$(1,-2)$	radius =	$\sqrt{5}$

$x-3 \geq 0$
$x-3=7$
$x=10$

$x-3<0$
$-(x-3)=7$
$-x+3=7$
$-x=4$
$x=-4$

$(x-2x+1)+(y+4y+4)=5$
$(x-1)^2+(y+2)^2=5$

2. Find the equation of the line through (3, 2) which is parallel to the line passing through (2, 1) and (5, 9).

Equation :	$y-2=\frac{8}{3}(x-3)$

$$y-2=\frac{\Delta y}{\Delta x}(x-3)$$
$$=\frac{9-1}{5-2}(x-3)$$
$$y-2=\frac{8}{3}(x-3)$$

3. Show that the equation $\frac{1}{x} = x^2 - 3x + 1$ has at least one solution on the interval (2, 3). Justify your answer.

$f(x) = \frac{1}{x} - x^2 + 3x - 1$ cont on $[2,3]$ because it is a polynomial plus a rational function + everywhere defined

$f(2) = \frac{1}{2} - 4 + 6 - 1 = \frac{3}{2} > 0$

$f(3) = \frac{1}{3} - 9 + 9 - 1 = -\frac{2}{3} < 0$

By the Root Location Theorem, there is a c in (2,3) with $f(c) = 0$, so $\frac{1}{c} - c^2 + 3c - 1 = 0$ and

$\frac{1}{c} = c^2 - 3c + 1$ ✓

4. Find the derivatives. You do not have to simplify your answers once you have finished differentiating.

a. $f(x) = \frac{1}{1+x^2}$.

$f'(x) =$	$-\frac{2x}{(1+x^2)^2}$

$$f'(x) = \frac{(1+x^2)\cdot(1)' - 1\cdot(1+x^2)'}{(1+x^2)^2} = \frac{0 - 2x}{(1+x^2)^2}$$

b. $g(t) = \frac{\ln(t)}{\cos t}$.

$g'(t) =$	

$$g'(t) = \frac{(\cos t)(\frac{1}{t}) - (\ln t)(-\sin t)}{\cos^2 t}$$

$$= \frac{\frac{1}{t}\cos t + \sin t \ln t}{\cos^2 t}$$

5. A person invests \$100 in a bank that compounds interest continuously. If the investment triples in 15 years, find the interest rate.

Interest rate equals:	$\frac{\ln 3}{15}$

$P(t) = 100e^{rt}$

$P(15) = 300 = 100e^{15r}$

$e^{15r} = 3$

$15r = \ln 3$

$r = \frac{\ln 3}{15}$

6. a. Find $\lim_{x\to 0} 2e^{-x^3}$.

Limit equals:	2

e^x is continuous everywhere, so just plug in

$2e^0 = 2$

b. Find the derivative of $g(x) = e^{\sin x}$.

$g'(x) =$	$(e^{\sin x})(\cos x)$

$\frac{d}{du} e^u = e^u \frac{du}{dx}$

$u = \sin x$

7a. Find $\lim_{x\to 2} \frac{x-2}{x^2-3x+2}$.

Limit =	1

$$\lim_{x\to 2} \frac{x-2}{(x-2)(x-1)} = \lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1} = 1$$

b. Find $\lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} 2(x+1) & \text{if } x < 3 \\ 4 & \text{if } x = 3. \\ x^2-1 & \text{if } x > 3 \end{cases}$

Limit =	6

The only suspicious point is $x=3$,* so we can find the limit by just plugging in $f(2) = 2(2+1) = 6$.

* So f is continuous at $x=2$

8. a. If $y = x^3(\sqrt{x}\sqrt[4]{x})^5$, find y'. $y' =$ $\frac{27}{4}x^{23/4}$

$y = x^3(x^{\frac{1}{2}} \cdot x^{\frac{1}{4}})^5 = x^3 \cdot (x^{3/4})^5 = x^3 \cdot x^{15/4} = x^{27/4}$

$y' = \frac{27}{4}x^{23/4}$

or

$y' = 3x^2(\sqrt{x}\sqrt[4]{x})^5 + x^3 \cdot 5(\sqrt{x}\sqrt[4]{x})^4\left[\frac{1}{2}x^{-\frac{1}{2}}\sqrt[4]{x} + \frac{1}{4}\sqrt{x}\,x^{-\frac{3}{4}}\right]$

8b. Find $\lim_{x\to 1} \frac{\frac{1}{x} - 1}{x}$.

Plug in $x = 1$, Get $\frac{\frac{1}{1} - 1}{1} = \frac{0}{1}$. This is defined, So the limit is $\frac{0}{1} = 0$

$\lim_{x\to 1} \frac{\frac{1}{x} - 1}{x} = 0$

9. An astronaut standing on the edge of a cliff on the planet Pogo jumps directly upward and observes that she passes her initial position on the way down exactly 4 seconds later. Three seconds after that, she hits the ground at the base of the cliff. What is her initial velocity? How high is the cliff? With what velocity does the rock hit the ground? $h(t) = -\frac{1}{2}gt^2 + v_0t + s_0$, and g = 2 ft/s^2 near planet Pogo's surface. Hint: What is $h(4)$? Plugging in gives you an equation that you can solve for v_0.

Initial velocity:	4

Cliff's height is:	21

Impact velocity is:	-10

$h(t) = -t^2 + v_0 t + s_0$ GIVEN INFORMATION

$\boxed{h(4) = h(0)}$, so

$-16 + 4v_0 + \cancel{s_0} = \cancel{s_0}$

$-16 + 4v_0 = 0$

$v_0 = 4$

$\boxed{h(7) = 0}$

$-7^2 + 4 \cdot 7 + s_0 = 0$

$-49 + 28 + s_0 = 0$

$s_0 = 21$

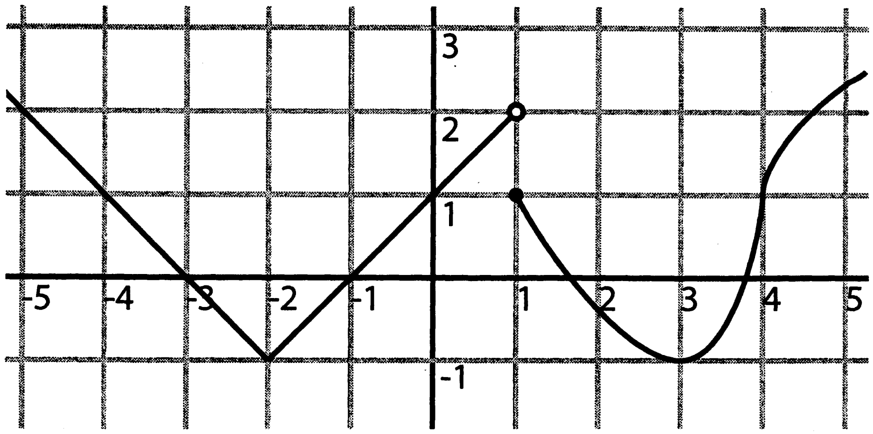
$v(t) = -2t + 4$

$v(7) = -14 + 4 = -10$

ALTERNATE QUESTION:

Suppose that when $t = 4$ she is 4 feet above her original ~~pos~~ position. Find the initial velocity, height of cliff, and impact velocity.

10. The graph of a function $f(x)$ is given below



Find all values of x in $[-5, 5]$ where f fails to be

a. continuous.

$x = 1$

b. differentiable.

$x = -2, 1, 4$

c. For which values of x is the derivative of f equal to 0?

$x = 3$

d. Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1+} f(x)$.

$\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = 1$