SPRING 2015 EXAM II

1. Find the equation of the tangent line to the curve $x^3 + y^3 = 2xy + 5$ at (1,2).

Equation:	$y-2=\frac{1}{10}(x-1)$

$$3x^2+3y^2y' = 2y+2xy'$$

$$3\cdot 1^2+3\cdot 2^2 y' = 2\cdot 2+2\cdot 1\cdot y'$$

$$3+12y' = 4+2y'$$

$$10y' = 1$$

$$y' = \frac{1}{10}$$

2. Find the absolute maximum and minimum of the function $f(x) = x + \dfrac{9}{x+2}$ on the interval $[-1, 2]$.

Absolute maximum:	$x=-1,\ f(-1)=8$
Absolute minimum:	$x=1,\ f(1)=4$

$$f'(x) = 1 - \frac{9}{(x+2)^2} = 0$$

$$(x+2)^2 = 9$$

$$x+2 = \pm 3$$

$\cancel{x=-5}$ NOT IN INTERVAL

$x=1$ CRIT #

$$f(-1) = -1+\frac{9}{1} = 8$$

$$f(1) = 1+\frac{9}{3} = 4$$

$$f(2) = 2+\frac{9}{4} = 4\tfrac{1}{4}$$

3. Find the c promised by the mean value theorem on the interval $[1, 3]$ if $f(x) = 1/x^2$. Remember to check the hypotheses of the mean value theorem.

c=	

$f(x) = \frac{1}{x^2}$ cont and diff for $x \neq 0$

$f'(x) = \frac{-2}{x^3}$

$$-\frac{2}{c^3} = \frac{\frac{1}{9} - \frac{1}{1}}{3-1} = \frac{-\frac{8}{9}}{2} = -\frac{4}{9}$$

$$\frac{c^3}{2} = \frac{9}{4} \qquad c^3 = 9/2 \qquad c = \sqrt[3]{9/2}$$

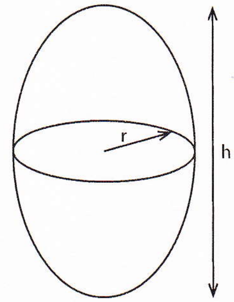
4. Find $\lim_{x \to 0} \frac{\sin x^2}{3x^2 + x^3}$.

$$\frac{0}{0} \lim_{x \to 0} \frac{2x \cos x^2}{6x + 3x^2}$$

$$\frac{0}{0} \lim_{x \to 0} \frac{2 \cos x^2 + (2x)(2x)(-\sin x^2)}{6 + 6x} = \frac{2}{6} = \frac{1}{3}$$

5. A manufacturer of chocolate eggs calculates that the volume of an egg of radius r and height h is $V = \frac{2\pi}{3}r^2h$. If company policy requires that $r + h = 10$, what is the maximum possible volume of a chocolate egg?

r:	$\frac{20}{3}$
h:	$\frac{10}{3}$
V:	$\frac{2\pi}{3}\left(\frac{20}{3}\right)^2\left(\frac{10}{3}\right)$



$V = \frac{2\pi}{3} r^2 h \qquad r + h = 10$

$h = 10 - r$

$V = \frac{2\pi}{3} r^2 (10 - r)$

$\frac{dV}{dr} = \frac{2\pi}{3}(20r - 3r^2) = \frac{2\pi}{3} r(20 - 3r) = 0$

$r = \frac{20}{3} \qquad h = 10 - \frac{20}{3} = \frac{10}{3}$

6. Air is being pumped into a spherical balloon so that its volume increases at a rate of 70 cm^3/s. How fast is the surface area of the balloon increasing when its radius is 11 cm? Recall that a ball of radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$.

$\frac{dS}{dt} =$	$\frac{140}{11}$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt} = 8\pi r \frac{dr}{dt}$$

$$70 = 4\pi \cdot 11^2 \cdot \frac{dr}{dt} \qquad \frac{dS}{dt} = \cancel{8}^{2}\cancel{\pi}\cancel{11} \cdot \frac{70}{\cancel{4}\cancel{\pi}\, 11^{\cancel{2}}}$$

$$\frac{dr}{dt} = \frac{70}{4\pi\, 11^2} \qquad = \frac{140}{11}$$

7. If $y = (2x+1)^{x^2}$, find $\frac{dy}{dx}$.

$$\ln y = \ln\left((2x+1)^{x^2}\right) = x^2 \ln(2x+1)$$

$$\frac{1}{y} y' = 2x \ln(2x+1) + x^2 \cdot \frac{1}{2x+1} \cdot 2$$

$$\boxed{y' = (2x+1)^{x^2}\left[2x\ln(2x+1) + \frac{2x^2}{2x+1}\right]}$$

8. Use linear approximation or differentials to approximate $\sqrt[3]{8.03}$.

$\sqrt[3]{8.03} \simeq$	$2 + \frac{.03}{12}$

$$L(x) = f(a) + f'(a)(x-a)$$

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$a = 8$$

$$x = 8.03$$

$$L(8.03) = 8^{\frac{1}{3}} + \frac{1}{3} 8^{\frac{-2}{3}}(8.03-8)$$

$$= 2 + \frac{1}{3} \cdot \frac{1}{4}(.03)$$

$$= 2 + \frac{.03}{12}$$

9. Find the horizontal and vertical asymptotes of the graph of $y = \dfrac{5e^x + 2}{3e^x - 3}$.

Horizontal asymptotes:	$5/3$, $-2/3$
Vertical asymptotes:	$x = 0$

$$\frac{\infty}{\infty} \quad \lim_{x \to \infty} \frac{5e^x + 2}{3e^x - 3} \overset{L'Hop}{=} \lim_{x \to \infty} \frac{5e^x}{3e^x} = \frac{5}{3}$$

$$\lim_{x \to -\infty} \frac{5e^x + 2}{3e^x - 3} = \frac{0 + 2}{0 - 3} = -\frac{2}{3}$$

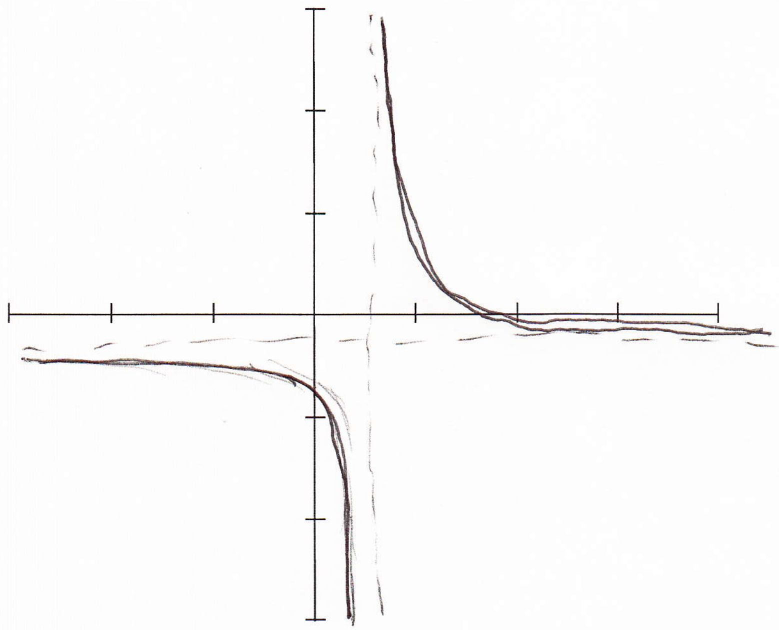
HA

VA $\quad 3e^x - 3 = 0$

$$e^x = 1$$

$$\underline{x = 0}$$

10. Sketch the graph of $f(x) = \dfrac{2-x}{6x-3}$, showing horizontal and vertical asymptotes, intervals where the function is increasing and decreasing, and intervals where the function is concave up and concave down. Hint: $f'(x) = -\dfrac{1}{(2x-1)^2}$ and $f''(x) = \dfrac{4}{(2x-1)^3}$.



Intervals where increasing:	NONE
Intervals where decreasing:	$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$
Intervals where concave up:	$(\frac{1}{2}, \infty)$
Intervals where concave down:	$(-\infty, \frac{1}{2})$
Horizontal asymptotes:	$y = -\frac{1}{6}$
Vertical asymptotes:	$x = \frac{1}{2}$
Inflections:	NONE

$$\frac{-\infty}{\infty} \quad \lim_{x \to \infty} \frac{2-x}{6x-3} \overset{\text{L'Hop}}{=} \lim_{x\to\infty} \frac{-1}{6}$$