EXAM II SOLUTIONS, FALL 2014

1. If $f(x) = \dfrac{1}{x}$, find the number c promised by the Mean Value Theorem on the interval [1, 3]. Don't forget to check the hypotheses of the MVT!

c =	$\sqrt{3}$

MVT If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there is a c in (a,b) so that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

$$f'(x) = -\frac{1}{x^2}$$

$$-\frac{1}{c^2} = \frac{\frac{1}{3}-1}{3-1} = \frac{-\frac{2}{3}}{2} = -\frac{1}{3}$$

$$c^2 = 3 \Rightarrow c = \sqrt{3}$$

$f(x) = \frac{1}{x}$ is differentiable on $(0, \infty)$.

2. Find the equation of the tangent line to the curve $x^2y^2 = x^2 + 2y^2 + 14$ at $(2, 3)$.

Equation:	$(y-3) = -\frac{8}{3}(x-2)$

$$2xy^2 + x^2 \cdot 2yy' = 2x + 4yy'$$

$$2 \cdot 2 \cdot 3^2 + 2^2 \cdot 2 \cdot 3y' = 2 \cdot 2 + 4 \cdot 3y'$$

$$36 + 24y' = 4 + 12y'$$

$$12y' = -32$$

$$y' = -\frac{32}{12} = -\frac{8}{3}$$

3a.	$d\left(\frac{\tan 2x}{3x}\right) =$	$\left(\frac{(3x)(\sec^2 2x)\cdot 2 - (\tan 2x)\cdot 3}{(3x)^2}\right)dx$

3b. If the cost of manufacturing q units of a product is $C(q) = 3q^2 + q + 300$, use marginal analysis to estimate the cost of producing the 17th item.

Cost =	97

$C'(q) = 6q + 1$

$C'(16) = 96 + 1 = 97$

4. a. Find $\lim\limits_{x\to\infty} \frac{e^{-2x}-2}{e^{-3x}+7}$.	$\lim\limits_{x\to\infty} \frac{e^{-2x}-2}{e^{-3x}+7} =$	$\frac{0-2}{0+7} = -\frac{2}{7}$
b. Find $\lim\limits_{x\to-\infty} \frac{e^{-2x}-2}{e^{-3x}+7}$.	$\lim\limits_{x\to-\infty} \frac{e^{-2x}-2}{e^{-3x}+7} =$	0

$$\lim_{x\to-\infty} \frac{e^{-2x}-2}{e^{-3x}+7} = \frac{\infty}{\infty}$$

$$\overset{\text{L'Hop}}{=} \lim_{x\to-\infty} \frac{-2e^{-2x}}{-3e^{-3x}} = \lim_{x\to-\infty} \frac{2}{3}e^{x} = \frac{2}{3}\cdot 0 = 0$$

5. Find the absolute maximum and minimum of the function $f(x) = \dfrac{4x}{x^2+4}$ on the interval $[1, 10]$. Please give both x and y values.

Absolute maximum:	$(2, 1)$
Absolute minimum:	$(10, \frac{40}{104})$

$$f'(x) = \frac{(x^2+4)\cdot 4 - (4x)(2x)}{(x^2+4)^2} = \frac{-4x^2+16}{(x^2+4)^2}$$

Crit #s $x = \pm 2$ only $x = 2$ is in $[1, 10]$.

$$f(1) = \frac{4}{5} = .8$$

$$f(2) = \frac{8}{8} = 1$$

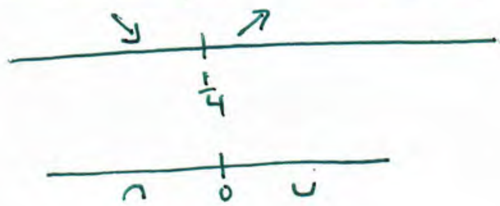
$$f(10) = \frac{40}{104} < .4$$

$f(x) = \dfrac{4x}{x^2+4}$ is a rational function that is defined everywhere, so it is continuous for all values of x.

This part was not required, but it's not a bad idea to think it through.

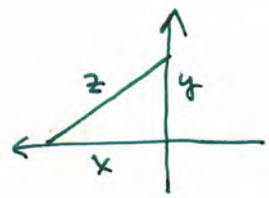
6. The function $f(x) = (2x-1)e^{4x}$ has $f'(x) = (8x-2)e^{4x}$ and $f''(x) = 32xe^{4x}$. Find the intervals where f is increasing and decreasing and concave up and concave down. Find the x-coordinates of all relative extrema.

Increasing:	$(0, \infty)$
Decreasing:	$(-\infty, \frac{1}{4})$
Concave up:	$(0, \infty)$
Concave down:	$(-\infty, 0)$
Relative Maxima:	NONE
Relative Minima:	$\frac{1}{4}$



7. At noon, a flatbed truck leaves Winslow, Arizona, traveling north at 65 miles per hour. At 2 pm, a Volkswagen bus leaves the same corner traveling west at 60 miles per hour. How fast is the distance between the two vehicles changing at 5 pm? You do not need to multiply out any big numbers.

Rate =	$\dfrac{180(60) + 325(65)}{\sqrt{180^2 + 325^2}}$



$x^2 + y^2 = z^2$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

$\frac{dz}{dt} = \dfrac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$

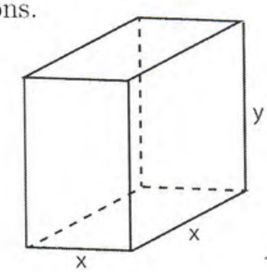
at 5 PM, $x = 180$, $y = 325$

$\frac{dx}{dt} = 60$, $\frac{dy}{dt} = 65$

$z = \sqrt{180^2 + 325^2}$

8. A rectangular parcel has a square base of side x and a third side of length y. Postal regulations say that the perimeter of the square plus the length of side y cannot exceed 102. Find the volume of the largest parcel allowed by these regulations.

Volume =	$17^2 \cdot 34$



$$4x + y = 102$$

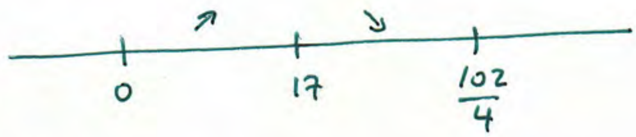
$$y = 102 - 4x$$

$$V = x^2 y = x^2(102 - 4x)$$

$$= 102x^2 - 4x^3$$

$$\frac{dV}{dx} = 204x - 12x^2 = 12x(17 - x)$$

CRIT #'S $x = 0,\ x = 17$ $\qquad 0 \le x \le \frac{102}{4}$



$$f(0) = 0$$

$$f(17) = 102 \cdot 17^2 - 4 \cdot 17^3 = 17^2(102 - 4 \cdot 17)$$

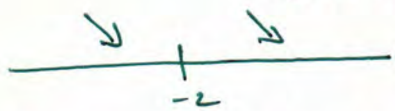
$$= 17^2(102 - 68)$$

$$= 17^2 \cdot 34$$

9. Let $f(x) = \dfrac{2x+5}{4x+2}$. Find intervals where f is increasing and decreasing, and concave up and concave down. Find all horizontal and vertical asymptotes and find all relative maxima, minima, and inflections. Sketch the graph for 1 pt extra credit.

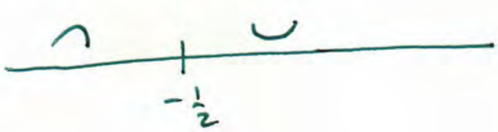
Intervals where increasing:	NONE
Intervals where decreasing:	$(-\infty,-2)\cup(-2,\infty)$
Intervals where concave up:	$(-\frac{1}{2},\infty)$
Intervals where concave down:	$(-\infty,-\frac{1}{2})$
Horizontal asymptotes:	$y=1/2$
Vertical asymptotes:	$x=-\frac{1}{2}$
Inflections:	NONE
Relative maxima:	NONE
Relative minima:	NONE

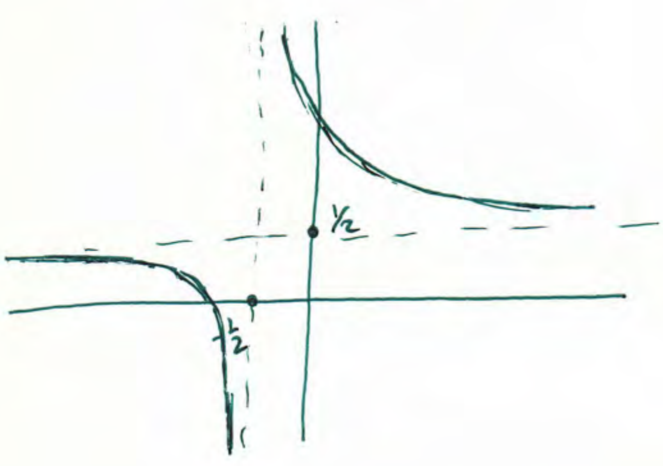
$$f'(x) = \frac{(4x+2)\cdot 2-(2x+5)\cdot 4}{(4x+2)^2} = \frac{4-20}{(4x+2)^2} = \frac{-16}{(4x+2)^2}$$



No crit #'s

$$f''(x) = \frac{(-16)(-2)}{(4x+2)^3}\cdot 4 = \frac{128}{(4x+2)^3}$$





10a. If $y = (\cos x)^{\sin x}$, find y'.

$y' =$	$(\cos x)^{\sin x}\left[\cos x \ln(\cos x) - \frac{\sin^2 x}{\cos x}\right]$

$$\ln y = \ln(\cos x)^{\sin x} = \sin x \ln(\cos x)$$

$$\frac{1}{y} y' = \cos x \ln(\cos x) + \sin x \cdot \frac{1}{\cos x} \cdot (-\sin x)$$

$$y' = (\cos x)^{\sin x}\left[\cos x \ln(\cos x) - \frac{\sin^2 x}{\cos x}\right]$$

10b. Use linear approximation or differentials to estimate $\sqrt{3.97}$.

$L(x) = f(a) + f'(a)(x-a)$

$f(x) = \sqrt{x}$ $a=4$, $x = 3.97$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f(a) = 2$

$f'(a) = \frac{1}{4}$

$L(3.97) = 2 + \frac{1}{4}(3.97-4)$

$$\boxed{= 2 + \frac{-.03}{4}}$$