ANSWER KEY EXAM #1 2015

1. (10 points) Find the equation of the tangent line to the curve with equation $y = x^2 + 7$ that is parallel to the line $2x - y - 3 = 0$.

Equation of line is:	

Slope of $2x - y - 3 = 0$ is 2. ($y = 2x - 3$)

$\frac{dy}{dx} = 2x$ slope of curve

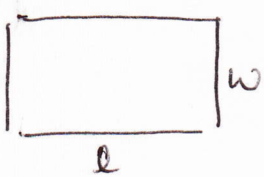
Slope of curve = slope of line $2x = 2$, $x = 1$

Point on curve is $(1, 8)$ so

$(y - 8) = 2(x - 1)$

2. If the area of a rectangle is 5 and its width is 3 less than twice its length, find the length of its diagonal.

Length of diagonal equals:	



Area $= lw = 5$

$w = 2l - 3$

$l(2l - 3) = 5$

$2l^2 - 3l = 5$

$2l^2 - 3l - 5 = 0$

$l = \frac{3 \pm \sqrt{9 + 40}}{4} = \frac{3 \pm 7}{4}$ $\frac{5}{2}$, ~~X~~

$w = 2(\frac{5}{2}) - 3 = 2$

diagonal $= \sqrt{(5/2)^2 + 2^2}$

3. Show that the equation $\sqrt[3]{x-1} = x^2 - 2$ has at least one solution on the interval (1, 2). Justify your answer.

$f(x) = \sqrt[3]{x-1} - x^2 + 2$ continuous because sum of polynomial and composition of polynomial + power function

$f(c) = 0 \Leftrightarrow c$ satisfies equation above

$f(1) = 0 - 1 + 2 > 0$

$f(2) = 1 - 4 + 2 < 0$

By IVT, there is a c in (1,2) with $f(c) = 0$.

4. Find the derivatives. You do not have to simplify your answers once you have finished differentiating.

a. $f(x) = \dfrac{\ln(x)}{1+5x^2}$.

$f'(x) =$	$\dfrac{(1+5x^2)\cdot\frac{1}{x} - \ln(x)\,10x}{(1+5x^2)^2}$

b. $g(t) = \sin(t^2)$.

$g'(t) =$	$(\cos t^2)(2t)$

5. The half-life of a sample of radium is 1590 years. If the sample contains 100 grams on February 26, 2015, how much will be left on February 26, 3015?

Amount of radium equals:	

$A(t) = 100e^{-kt}$

$A(1590) = 50 = 100e^{-k\cdot 1590}$

$\frac{1}{2} = e^{-k\cdot 1590}$

$\ln(\frac{1}{2}) = -k\cdot 1590$

$k = -\frac{\ln\frac{1}{2}}{1590}$

$\rightarrow A(1000) = 100e^{+\frac{\ln(\frac{1}{2})}{1590}\cdot 1000}$

6. a. Use the algebraic techniques to find $\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$.

Limit equals:	$1/10$

$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} \cdot \frac{\sqrt{x}+5}{\sqrt{x}+5} = \lim_{x\to 25} \frac{\cancel{x-25}}{(x-25)(\sqrt{x}+5)} = \lim_{x\to 25} \frac{1}{\sqrt{x}+5} = \frac{1}{10}$

b. Find the derivative of $g(x) = \frac{e^2}{1-\sin(x)}$.

$g'(x) =$	$\frac{e^2\cos x}{(1-\sin x)^2}$

$g'(x) = \frac{(1-\sin x)\cdot 0 - e^2\cdot(-\cos x)}{(1-\sin x)^2}$

7a. Find $\lim_{x \to 5} \frac{x-5}{x^2-6x+5}$. **Limit =** $1/4$

$$\lim_{x \to 5} \frac{(x-5)}{(x-5)(x-1)} = \lim_{x \to 5} \frac{1}{x-1} = \frac{1}{4}$$

b. For what values of c is the function

$$f(x) = \begin{cases} -2x+c & x \le 3 \\ \dfrac{-4}{x-c} & x > 3 \end{cases}$$

continuous for all values of x?

$$\lim_{x \to 3^-} f(x) = -2 \cdot 3 + c$$

$$\lim_{x \to 3^+} f(x) = \frac{-4}{3-c}$$

$\lim_{x \to 3} f(x)$ exists when $-6+c = \frac{-4}{3-c}$

$$(-6+c)(3-c) = -4$$

$$-c^2+9c-18 = -4$$

$$c^2-9c+14 = 0$$

$$(c-7)(c-2) = 0$$

$c = 2, \cancel{7}$

$c = 7$ doesn't work because f is discontinuous at $x = 7$.

(This is a webwork problem.)

8. a. If $y = \dfrac{x^4}{(\sqrt[3]{x})^3}$, find y'.

$y' =$	$3x^2, \ x \neq 0$

$$y = \frac{x^4}{x} = x^3, \quad x \neq 0$$

$$y' = 3x^2, \quad x \neq 0$$

I'd be happy with
$y = 3x^2$ on this one!

8b. If $f(x) = x^3 h(x)$ with $h(-2) = 3$ and $h'(-2) = 4$, calculate $f'(-2)$.

$$f'(x) = x^3 h'(x) + 3x^2 h(x)$$

$$f'(-2) = (-2)^3 \cdot h'(-2) + 3(-2)^2 h(-2)$$

$$= -8 \cdot 4 + 3 \cdot 4 \cdot 3 = -32 + 36 = 4$$

9. An astronaut standing on the edge of a cliff on the planet Pogo jumps directly upward and observes that on the way down she passes a point one foot above her initial position exactly 4 seconds later. * Four seconds after that, she hits the ground at the base of the cliff. ** What is her initial velocity? How high is the cliff? $h(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$, and g = 2 ft/s^2 near planet Pogo's surface. Hint: What is $h(4)$? Plugging in gives you an equation that you can solve for v_0.

Initial velocity:	$17/4$

Cliff's height is:	30

$h(t) = -t^2 + v_0 t + s_0$ (from g = 2)

$h(4) = h(0) + 1$ From *.

$h(8) = 0$ From **.

$-4^2 + v_0 \cdot 4 + \cancel{s_0} = -0^2 + v_0 \cdot 0 + \cancel{s_0} + 1$

$-16 + 4v_0 = 1$

$4v_0 = 17$

$v_0 = 17/4$

$-8^2 + 8v_0 + s_0 = 0$

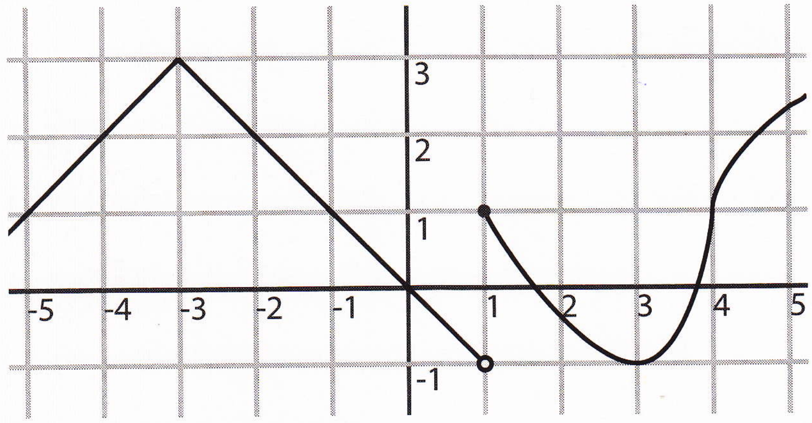
$-8^2 + 8 \cdot \frac{17}{4} + s_0 = 0$

$-64 + 34 + s_0 = 0$

$-30 + s_0 = 0$

$s_0 = 30$

10. The graph of a function $f(x)$ is given below



Find all values of x in $[-5, 5]$ where f fails to be

a. continuous.

$x = 1$

b. differentiable.

$x = -3, 1, 4$

c. For which values of x is the derivative of f equal to 0?

$x = 3$

d. Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

$\lim_{x \to 1^-} f(x) = -1$

$\lim_{x \to 1^+} f(x) = +1$