EXAM II Nov. 14, 2013

1. If $f(x) = x^3 + 4x$, find the number c promised by the Mean Value Theorem on the interval [1, 2].

c =	$\sqrt{7/3}$

$$f'(c) = \frac{f(2) - f(1)}{2 - 1} = \frac{16 - 5}{1}$$

$$3c^2 + 4 = 11$$

$$3c^2 = 7$$

$$c = \sqrt{7/3}$$ (THE NEGATIVE ROOT IS NOT IN THE INTERVAL.)

2. Find the equation of the tangent line to the curve $x^3 + y^3 = x^2 + y^2 + 4$ at $(2, 1)$.

Equation:	$(y - 1) = -8(x - 2)$

$$3x^2 + 3y^2y' = 2x + 2yy'$$

$$3 \cdot 4 + 3 \cdot 1 \cdot y' = 2 \cdot 2 + 2 \cdot 1 \cdot y'$$

$$12 + 3y' = 4 + 2y'$$

$$y' = -8$$

3. Let $f(x) = \dfrac{x^3}{x^2+1}$. Then $f'(x) = \dfrac{x^2(x^2+3)}{(x^2+1)^2}$.

   a. Compute $f(2)$.

   b. Compute $f'(2)$.

   c. Using the differential or tangent line approximation, find an approximate value for $f(2.03)$. You do not need to simplify your answer.

$f(2) =$	$8/5$
$f'(2) =$	$28/25$
$f(2.03) \approx$	$8/5 + (28/25)(.03)$

$f(2) = \dfrac{2^3}{2^2+1} = \dfrac{8}{5}$

$f'(2) = \dfrac{4(4+3)}{(2^2+1)^2} = \dfrac{28}{25}$

4. a. Find $\displaystyle\lim_{x\to 0} \frac{x^2}{\sqrt{9+5x^2}-3}$. $\overset{\frac{0}{0}\ \text{L'Hop}}{=} \displaystyle\lim_{x\to 0} \frac{2\cancel{x}}{\frac{1}{2}(9+5x^2)^{-\frac{1}{2}}\cdot 10\cancel{x}} = \frac{2}{\frac{1}{2}(9)^{-\frac{1}{2}}\cdot 10} = \frac{6}{5}$

   b. Find $\displaystyle\lim_{x\to\infty} \frac{e^{-x}+5}{e^{-x}+7}$. $= \dfrac{0+5}{0+7} = \dfrac{5}{7}$

In 4a, you can use L'Hop again and get the answer, provided that you do the algebra correctly. You can also multiply by the conjugate like we did back toward the beginning of the course.

5. Find the absolute maximum and minimum of the function $f(t) = \dfrac{4x}{x^2+4}$ on the interval $[-4, 1]$.

Absolute maximum:	$(1, 4/5)$
Absolute minimum:	$(-2, -1)$

$$f'(x) = \frac{(x^2+4)\cdot 4 - 4x(2x)}{(x^2+4)^2} = \frac{4x^2+16-8x^2}{(x^2+4)^2}$$

$$= \frac{16-4x^2}{(x^2+4)^2} = \frac{4(4-x^2)}{(x^2+4)^2}$$

Crit #s: $x = \pm 2$, $x = -2$ in interval

$f(-4) = \frac{-16}{16+4} = -\frac{4}{5}$

$f(-2) = \frac{-8}{4+4} = -1$

$f(1) = 4/5$

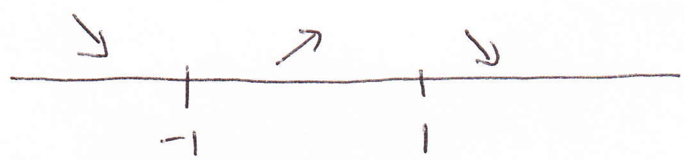
6. Find the intervals where the function $f(x) = e^{-x}(x+1)^2$ is increasing and decreasing. Find the x-coordinates of all relative extrema.

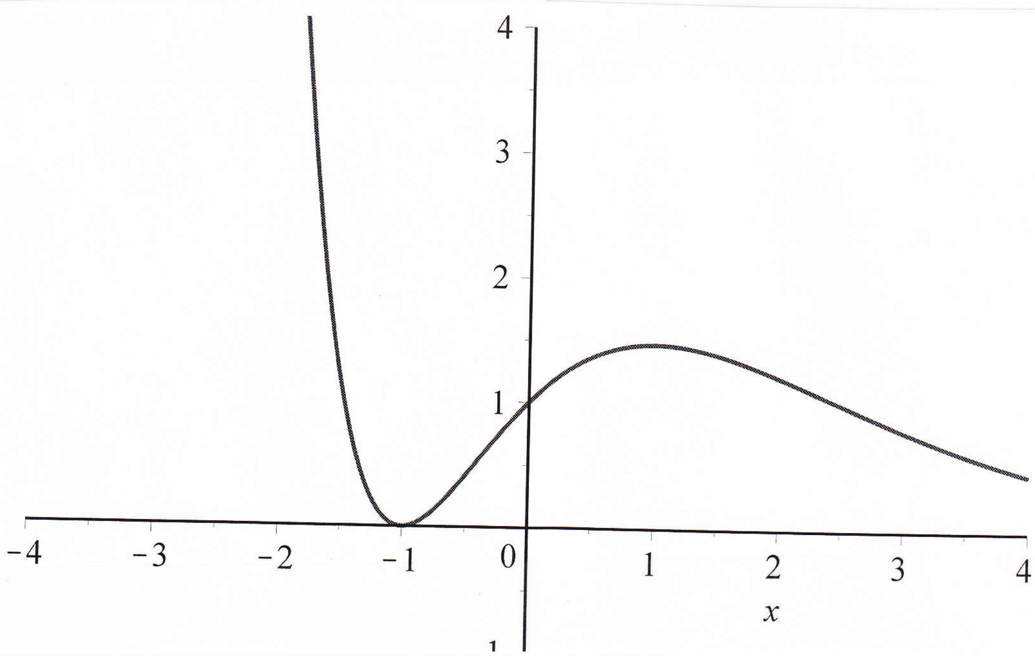
Increasing:	$(-1, 1)$
Decreasing:	$(-\infty, -1) \cup (1, \infty)$
Relative Maxima:	$x = 1$
Relative Minima:	$x = -1$

$$f'(x) = -e^{-x}(x+1)^2 + e^{-x} \cdot 2(x+1)$$

$$= e^{-x}(x+1)\left[-(x+1) + 2\right]$$

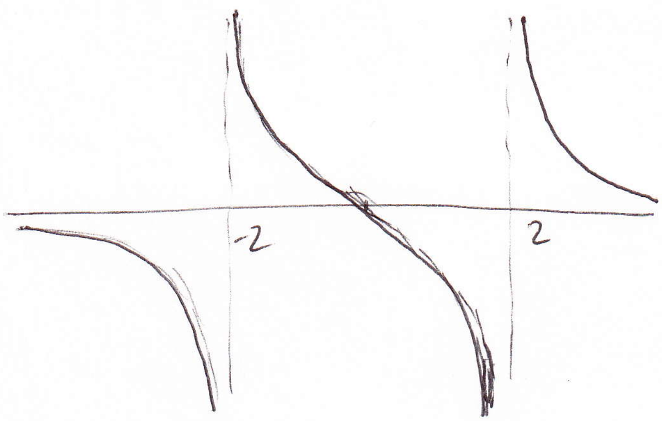
$$= e^{-x}(x+1)(-x+1) = -e^{-x}(x+1)(x-1)$$





7. Find the intervals where the function $f(x) = \dfrac{x}{x^2-4}$ is increasing and decreasing and concave up and concave down. Find all horizontal asymptotes, vertical asymptotes, and inflections. Hint: $f'(x) = -\dfrac{(x^2+4)}{(x^2-4)^2}$ and $f''(x) = \dfrac{2x(x^2+12)}{(x^2-4)^3}$.

Intervals where increasing:	NONE
Intervals where decreasing:	$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$
Intervals where concave up:	$(-2,0)\cup(2,\infty)$
Intervals where concave down:	$(-\infty,-2)\cup(0,2)$
Horizontal asymptotes:	$y=0$
Vertical asymptotes:	$x=\pm 2$
Inflections:	$x=0$



8. The surface area of a sphere is decreasing at the constant rate of 5π cm^2 per second. At what rate is the volume of the sphere decreasing at the instant its radius is 3 cm? $V = \frac{4}{3}\pi r^3$ (volume) and $S = 4\pi r^2$ (surface area).

$$\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$$

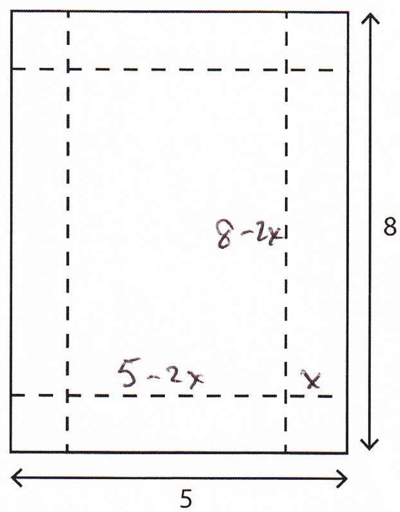
$$-5\pi = 8\pi\cdot 3\frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{5}{24}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} = 4\pi\cdot\overset{3}{\cancel{9}}\cdot\left(-\frac{5}{\underset{2}{\cancel{24}}}\right)$$

$$= -\frac{15\pi}{2}$$

9. A tinsmith wants to make an open-topped box out of a rectangular sheet of tin 5 inches wide and 8 inches long. The tinsmith plans to cut squares out of each corner of the sheet and then bend the edges of the sheet upward to form the sides of the box. For what dimensions does the box have the greatest possible volume?

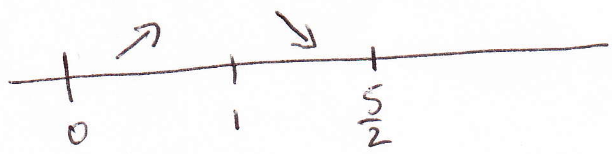


$$V = (5-2x)(8-2x)x$$
$$= (40-26x+4x^2)x$$
$$= 40x-26x^2+4x^3$$
$$\frac{dV}{dx} = 40-52x+12x^2$$
$$= 4(10-13x+3x^2)$$
$$= 4(3x-10)(x-1) = 0$$
$x=1$, $x=\frac{10}{3}$ crit #s

$5-2\left(\frac{10}{3}\right) < 0$ so IMPOSSIBLE

$\boxed{x=1}$ $\boxed{6\times 3\times 1}$

If you want to check that it's a max



10. If $y = x^{\tan(x)}$, find y'.

$\ln y = \ln x^{\tan x} = \tan x \ln x$

$\frac{1}{y} y' = \sec^2 x \ln x + \frac{\tan x}{x}$

$y' = x^{\tan x}\left[\sec^2 x \ln x + \frac{\tan x}{x}\right]$