EXAM II FALL 2014

1. If $f(x) = \dfrac{1}{x}$, find the number c promised by the Mean Value Theorem on the interval $[1, 3]$. Don't forget to check the hypotheses of the MVT!

c =	$\sqrt{3}$

f(x) is a rational function, continuous and differentiable for $x \neq 0$.

$$f'(c) = \frac{f(3)-f(1)}{3-1} = \frac{\frac{1}{3}-1}{2} = \frac{-\frac{2}{3}}{2} = -\frac{1}{3}$$

$f'(x) = -\frac{1}{x^2}$ so $-\frac{1}{c^2} = -\frac{1}{3}$ $\quad c = \pm\sqrt{3}$

The positive $\sqrt{3}$ is in $[1,3]$

2. Find the equation of the tangent line to the curve $x^2y^2 = x^2 + 2y^2 + 14$ at $(2, 3)$.

Equation:	$y - 3 = -\frac{8}{3}(x-2)$

$$2xy^2 + x^2 \cdot 2yy' = 2x + 4yy'$$

$$2\cdot2\cdot3^2 + 2^2\cdot2\cdot3\,y' = 2\cdot2 + 4\cdot3\cdot y'$$

$$36 + 24y' = 4 + 12y'$$

$$12y' = -32$$

$$y' = -\frac{32}{12} = -\frac{8}{3}$$

3a.	$d\left(\frac{\tan 2x}{3x}\right)=$	$\frac{3x\cdot(\sec^2 2x)\cdot 2-(\tan 2x)\cdot 3}{9x^2}\,dx$

3b. If the cost of manufacturing q units of a product is $C(q) = 3q^2 + q + 300$, use marginal analysis to estimate the cost of producing the 17th item.

Cost =	97

$MC(q) = C'(q) = 6q + 1$

$MC(16) = 6\cdot 16 + 1 = 97$

4. a. Find $\lim\limits_{x\to\infty} \frac{e^{-2x}-2}{e^{-3x}+7}$.

$\lim\limits_{x\to\infty} \frac{e^{-2x}-2}{e^{-3x}+7} =$	$\frac{-2}{7}$

b. Find $\lim\limits_{x\to-\infty} \frac{e^{-2x}-2}{e^{-3x}+7}$.

$\lim\limits_{x\to-\infty} \frac{e^{-2x}-2}{e^{-3x}+7} =$	0

$\lim\limits_{x\to\infty} e^{-2x} = \lim\limits_{x\to\infty} e^{-3x} = 0$

$\lim\limits_{x\to-\infty} e^{-2x} = \lim\limits_{x\to-\infty} e^{-3x} = \infty$

$\lim\limits_{x\to-\infty} \frac{e^{-2x}-2}{e^{-3x}+7} \overset{\text{L'HOP}}{=} \lim\limits_{x\to-\infty} \frac{-2e^{-2x}}{-3e^{-3x}} = \lim\limits_{x\to-\infty} \frac{2}{3}e^{x}$

$= 0$

5. Find the absolute maximum and minimum of the function $f(x) = \dfrac{4x}{x^2+4}$ on the interval $[1, 10]$. Please give both x and y values.

Absolute maximum:	$x=2,\ y=1$
Absolute minimum:	$x=10,\ y=\frac{40}{104}$

$$f'(x) = \frac{(x^2+4)\cdot 4 - 4x(2x)}{(x^2+4)^2} = \frac{4x^2+16-8x^2}{(x^2+4)^2}$$

$$= \frac{-4x^2+16}{(x^2+4)^2}$$

$f'(x)=0$ when $-4x^2+16=0$

$$x^2=4$$

$$x=\pm 2$$

crit # is $x=2$.

$$f(1) = \frac{4}{5}$$

$$f(2) = \frac{8}{8} = 1$$

$$f(10) = \frac{40}{104} = \frac{5}{13}$$

6. The function $f(x) = (2x-1)e^{4x}$ has $f'(x) = (8x-2)e^{4x}$ and $f''(x) = 32xe^{4x}$. Find the intervals where f is increasing and decreasing and concave up and concave down. Find the x-coordinates of all relative extrema.

Increasing:	$[\frac{1}{4}, \infty)$
Decreasing:	$(-\infty, \frac{1}{4}]$
Concave up:	$[0, \infty)$
Concave down:	$(-\infty, 0]$
Relative Maxima:	NONE
Relative Minima:	$x = \frac{1}{4}$ $\searrow\nearrow$

$f'(x) > 0$ when $8x-2 > 0 \iff x > \frac{1}{4}$

$f''(x) > 0$ when $x > 0$

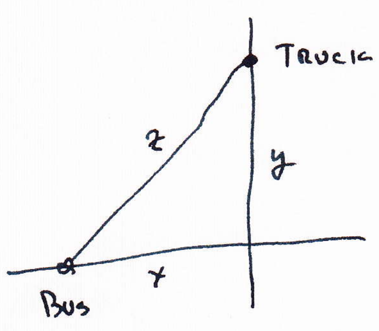
7. At noon, a flatbed truck leaves Winslow, Arizona, traveling north at 65 miles per hour. At 2 pm, a Volkswagen bus leaves the same corner traveling west at 60 miles per hour. How fast is the distance between the two vehicles changing at 5 pm? You do not need to multiply out any big numbers.

Rate =	$\dfrac{180\cdot 60 + 325\cdot 65}{\sqrt{325^2+180^2}}$

$$z^2 = x^2 + y^2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$



AT 5PM:

$y = 5\cdot 65 = 325$

$x = 3\cdot 60 = 180$

$\frac{dy}{dt} = 65$

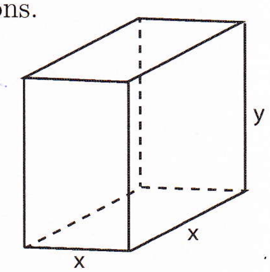
$\frac{dx}{dt} = 60$

$z = \sqrt{325^2 + 180^2}$

$\Rightarrow \dfrac{dz}{dt} = \dfrac{180\cdot 60 + 325\cdot 65}{\sqrt{325^2+180^2}}$

8. A rectangular parcel has a square base of side x and a third side of length y. Postal regulations say that the perimeter of the square plus the length of side y cannot exceed 102. Find the volume of the largest parcel allowed by these regulations.

Volume =	$17^2 \cdot 34$



$4x + y = 102$ ← POSTAL REGULATION

$V = x^2 y$

$y = 102 - 4x$

$V = x^2(102 - 4x) = 102x^2 - 4x^3$

$\frac{dV}{dx} = 204x - 12x^2 = x(204 - 12x) = 0$

$x = 0, \quad x = \frac{204}{12} = 17$

$0 \le x \le \frac{102}{4} = \frac{17}{2}$

$V(0) = 0$

$V(17) = 17^2 \cdot (102 - 4 \cdot 17) = 17^2 \cdot 34$

9. Let $f(x) = \dfrac{2x+5}{4x+2}$. Find intervals where f is increasing and decreasing, and concave up and concave down. Find all horizontal and vertical asymptotes and find all relative maxima, minima, and inflections. Sketch the graph for 1 pt extra credit.

Intervals where increasing:	NONE
Intervals where decreasing:	$(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$
Intervals where concave up:	$(-\frac{1}{2}, \infty)$
Intervals where concave down:	$(-\infty, -\frac{1}{2})$
Horizontal asymptotes:	$y = \frac{1}{2}$
Vertical asymptotes:	$x = -\frac{1}{2}$
Inflections:	NONE
Relative maxima:	NONE
Relative minima:	NONE

$$f'(x) = \frac{(4x+2)\cdot 2 - (2x+5)\cdot 4}{(4x+2)^2} = \frac{8x+4-8x-20}{(4x+2)^2} = \frac{-16}{(4x+2)^2}$$

$x = -\frac{1}{2}$

$$f''(x) = \frac{-16}{(4x+2)^3}\cdot(-2)\cdot 4 = \frac{32\cdot 4}{(4x+2)^3}$$

$$\lim_{x\to\infty} f(x) = \frac{1}{2}$$

$$\lim_{x\to-\infty} f(x) = \frac{1}{2}$$

10a. If $y = (\cos x)^{\sin x}$, find y'.

$y' =$	$(\cos x)^{\sin x}\left(\cos x \ln \cos x - \frac{\sin^2 x}{\cos x}\right)$

$$\ln y = \sin x \ln \cos x$$

$$\frac{1}{y} y' = \cos x \ln \cos x + \sin x \cdot \frac{1}{\cos x} \cdot (-\sin x)$$

$$y' = y\left(\cos x \ln \cos x - \frac{\sin^2 x}{\cos x}\right)$$

$$y' = (\cos x)^{\sin x}\left(\cos x \ln \cos x - \frac{\sin^2 x}{\cos x}\right)$$

10b. Use linear approximation or differentials to estimate $\sqrt{3.97}$.

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 2 + \frac{1}{4}(x-4)$$

$$L(3.97) = 2 + \frac{1}{4}(-.03)$$

$$\boxed{2 + \frac{-.03}{4} = 1.9925}$$

$f(x) = \sqrt{x}$

$a = 4$

$x = 3.97$

$f'(x) = \frac{1}{2}\frac{1}{\sqrt{x}}$

$f(a) = \sqrt{4} = 2$

$f'(a) = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} = \frac{1}{4}$