Name _____ MA135 Final Exam A December 15, 2014

Instructor _____ Section _____

Be sure to show all of your work. All solutions should use calculus techniques from this course. Unsupported answers will receive no credit! Calculators are not allowed on this exam. You may only use the formula sheet and scratch paper supplied with this exam. Good Luck!!

Prob No.	Max Pts	Points	Prob No.	Max Pts	Points
1	18		8	18	
2	18		9	18	
3	18		10	18	
4	18		11	18	
5	17		12	18	
6	18		13	18	
7	17		14	18	
Subtotal	125		**Subtotal**	125	

Grand Total	

2

1. (9 points each) You do not need to simplify your answers.

a. Find $\dfrac{d}{dx}\sin(5x^2)$.

$\dfrac{d}{dx}\sin(5x^2) =$	

b. A frog hops out of a pool of radioactive goo and into a lab. The scientists discover that the frog registers 10 Curies of radioactivity initially and that three days later it registers 7 Curies of radioactivity. When will its level of radioactivity reach 2 Curies? This is an exponential decay problem.

Time to reach 2 Curies =	

2. (9 points each)

a. Find the equation of the tangent line to

$$x^3 + y^3 - xy = 7$$

at $(2, 1)$. You may use any form of the equation of a line.

The line is:	

b. Find $\lim\limits_{x \to -1} \dfrac{x^{10} - 1}{x + 1}$.

$\lim\limits_{x \to -1} \dfrac{x^{10} - 1}{x + 1} =$	

3. (9 points each) You do not need to simplify your answers.

a. Find $\displaystyle\int_1^4 \frac{x^2 + 2\sqrt{x}}{x}\,dx.$ $\displaystyle\int_0^1 \frac{x^2 + 2\sqrt{x}}{x}\,dx \;=\;$

b. Find $\displaystyle\lim_{x \to -\infty} \frac{e^x + 7}{2e^x + 5}.$ $\displaystyle\lim_{x \to -\infty} \frac{e^x + 7}{2e^x + 5} \;=\;$

4.

a. (9 points) Approximate $\int_0^1 \sqrt{1-x^3}\, dx$ using 5 rectangles of equal width and right end-points. You do not need to simplify your answer.

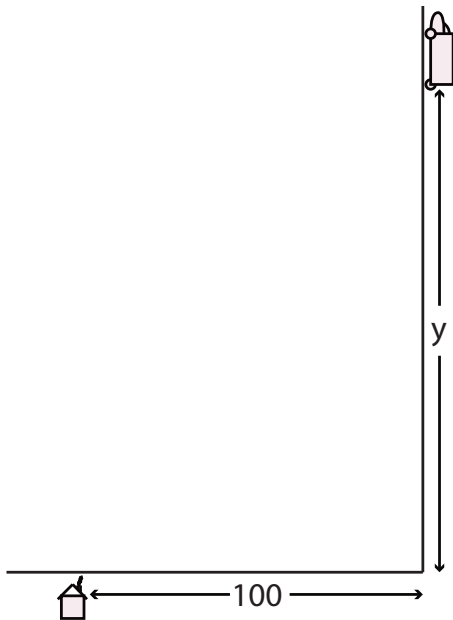| $\int_0^1 \sqrt{1-x^3}\, dx \doteq$	

b. (9 points) If $G(x) = \int_0^x e^{-t^2}\, dt$, find $G'(x)$.

$G'(x) =$	

5. (17 points) At noon, a truck heads north from an intersection at a constant speed of 60 miles per hour. There is a house 100 miles to the west of the intersection. How fast is the distance between the truck and the house changing at 3 pm? You do not need to simplify your answer.

Rate of change =	

y

100

6. (18 points) The marginal revenue of a certain commodity is $R'(x) = -3x^2 + 4x + 32$, where x is the level of production in thousands. Assume $R(0) = 0$. Find $p(x)$ (the demand function).

$p(x) =$	

7. (17 points) Find the x-values of the absolute maxima and minima of the function
$$f(x) = \frac{16}{x} + x^2 - 10$$
on the interval $[1, 4]$. (You must use calculus methods from this course to solve this problem. Plugging in numbers and guessing will receive no credit.)

Absolute max at x =	
Absolute min at x=	

8. (18 points) The wicked queen is selling apples to her wicked friends at a price of $7 per apple and at this price her friends have been buying 45 apples per month. She gets apples for free (don't ask!), but the cost of eye of newt and other evil ingredients is $2 per apple. She wishes to raise the price and estimates that for each $1 increase in price, 3 fewer apples will be sold each month. At which price should she sell the apples so as to maximize her wicked profit?

Price per apple =	

9. (18 pts) Find the intervals where the function $y = \dfrac{x^3}{x^2 - 1}$ is increasing and decreasing, concave up, and concave down. Find all horizontal, vertical asymptotes, relative extrema, and inflections. Write "none" in the blank if there are none. Hint: $y' = \dfrac{x^2(x^2 - 3)}{(x^2 - 1)^2}$ and $y'' = \dfrac{2x(x^2 + 3)}{(x^2 - 1)^3}$. Also, $\sqrt{3} \doteq 1.7$.

Increasing	
Decreasing	
Concave up	
Concave down	
Horizontal asymptotes	
Vertical asymptotes	
x-values of relative maxima	
x-values of relative minima	
x-values of inflections	

10. (9 points each)

a. Suppose that $S = \frac{16}{15}\pi^3 r^6$. If there is an error of 3 percent in the measurement of r, approximately what percentage error should be expected in the computation of S?[1]

Percentage error =	

b. Find the exact area under the curve $y = \dfrac{1}{\sqrt{x^3}}$ from $x = 3$ to $x = 5$. You do not need to simplify your answer.

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[1] In case you're curious where the strange formula came from, Zaphod Beeblebrox is hyperpainting the spherical hyperdrive on his 7-dimensional transluminal racer. He needs to know how much extra hyperpaint to buy, since he's too busy being president of the galaxy to measure r with better than 3 percent accuracy. This information is neither necessary nor useful in solving problem 10a.

11. (18 points) A tinsmith wants to make an open-topped box out of a rectangular sheet of tin 3 in wide and 8 in long. The tinsmith plans to cut congruent squares out of each corner of the sheet and then bend and solder the edges of the sheet upward to form the sides of the box, as shown below. For what value of x does the box have the greatest possible volume? You do not have prove that your answer is a maximum.

x =
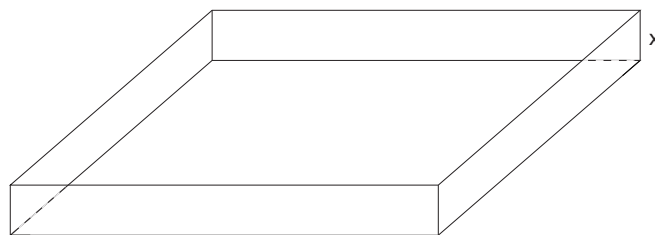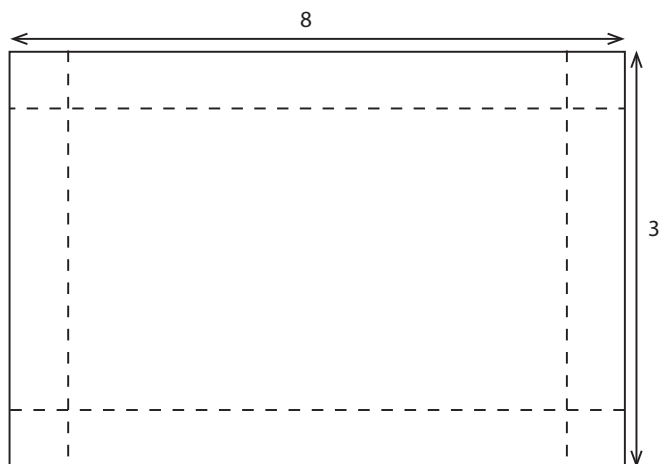
12. (18 points) Find $\displaystyle\int_{-7}^{3} f(x)\, dx$ where $f(x) = \begin{cases} x & \text{if } x < 1 \\ \dfrac{1}{x} & \text{if } x \geq 1 \end{cases}$.

13. (18 points) Find $\displaystyle\int \frac{dx}{x \ln x}$.

14. (9 points each)

   a. Find $\displaystyle\int_1^2 (2x+9)^5 \, dx.$ $\boxed{\displaystyle\int_1^2 (2x+9)^5 \, dx =}$

   You do not need to simplify this answer!

   b. If $f(x) = (1+x^2)^{\sin x}$, find $f'(x)$. $\boxed{f'(x)=}$