Practice test for exam #1 in MA135

In addition to problems on this sheet, students should study webwork problems, assigned homework, worked examples in the text, review problems at the end of each chapter, and Prof Sims' sample exams on the web.

$= \frac{1}{4}$

1. Find $\lim\limits_{x\to 2} \dfrac{\sqrt{x+2}-2}{x-2}$ or harder, $\lim\limits_{x\to 0} \dfrac{1-\frac{1}{x+1}}{x}$. $= 1$

2. Find $\lim\limits_{x\to 3} f(x)$ if

$\lim\limits_{x\to 3} f(x) = 8$

$$f(x) = \begin{cases} 2(x+1) & if\ x < 3 \\ 4 & if\ x = 3 \\ x^2 - 1 & if\ x > 3. \end{cases}$$

3. Prove that the equation $\cos x - \sin x = x$ has at least one solution on the interval $(0,\ \frac{\pi}{2})$.

$= 3/5$

4. Evaluate $2^{\log_2 3 - \log_2 5}$. Harder – A certain bank pays 6% interest compounded continuously. How long will it take for \$835 to double? The answer to this last question is $\dfrac{\ln 2}{.06}$. You can leave the answer in this form.

$f(x) = x^3$ $x$

5. Find the equation for the tangent line to the graph of the function $\cancel{f(s) = s^3}$ at $\cancel{s} = 1/2$. At what point(s) on the curve is the tangent line parallel to the line $2x - 3y = 7$? $x =$

$(y - \frac{1}{6}) = \frac{3}{4}(x - \frac{1}{2})$, $x = \pm \sqrt{2}/3$

6. Find $a$, $b$, $c$ so that the function $f(x) = ax^2 + bx + c$ will have $x$-intercepts at $(0,\ 0)$ and $(5,\ 0)$ and a tangent line with slope 1 where $x = 2$. $a = -1$, $b = 5$, $c = 0$

7. Find the derivative of $f(\theta) = \dfrac{\theta - 1}{2 + \cos\theta}$. Find the second derivative of $g(t) = t^3 e^t$.

$f'(\theta) = \dfrac{(2+\cos\theta)\cdot 1 - (\theta-1)(-\sin\theta)}{(2+\cos\theta)^2}$

8. Suppose a person standing on top of theTower of Pisa (176 ft high) throws a ball directly upward with an initial speed of 96 ft/s.

   a. Find the ball's height, its velocity, and acceleration at time $t$.
   b. When does the ball hit the ground and what is its impact velocity?
   c. How far does the ball travel during its flight?

Remark: On a test, I'd have to rework this so that students wouldn't need a calculator.

$h(t) = -16t^2 + 96t + 176$ $v(t) = -32t + 96$ $a = -32$

$h(t) = 0$ when $t = 3 + 2\sqrt{5}$

It travels 464 ft.

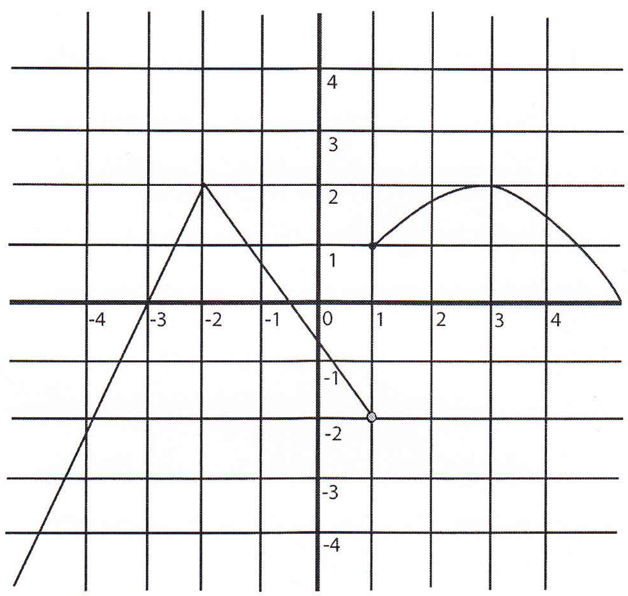
9. A manufacturer of light bulbs estimates that the fraction $F(t)$ of bulbs that remain burning after $t$ weeks is given by $F(t) = e^{-kt}$ where $k$ is a positive constant. Suppose twice as many bulbs are burning after 5 weeks as after 9 weeks.

a. Find $k$ and determine the fraction of bulbs still burning after 7 weeks. $k = \frac{1}{4}\ln 2, \ \frac{1}{4} 2^{1/4}$

b. What fraction of the bulbs burn out before 10 weeks? $1 - \frac{1}{8}\sqrt{2}$

c. What fraction of the bulbs can be expected to burn out between the 4th and 5th weeks? $F(4) - F(5) = \frac{1}{2} - \frac{1}{4} 2^{3/4}$

10. The graph of a function $f(x)$ is given below




Find all values of x where $f$ fails to be

a. continuous. $x = 1$

b. differentiable. $x = -2, x = 1$

c. For which values of x is the derivative of $f$ equal to 0? $x = 3$