Review Problems for the first exam in Math 135 Spring 2009.
NOTE : These are only practice problems!

The exam will cover all the material through section 3.5.

1. Find the equation of the line that passes through $(2,4)$ and is perpendicular to the line $2x+3y=12$.

2. Let $f(x)=\sqrt{x^2+2x-15}\,,\quad g(x)=\dfrac{1}{x}$

a) Find the domain of the function $f(x)$.
b) Find $g(f(x))$ and $f(g(x))$.
c) Find the domains of $g(f(x))$.

3. Find the **exact** value of each of the following limits. Show all work and/or give reasons for your answers:

a) $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-4}$ b) $\lim\limits_{x\to 4}\dfrac{x^2-5x+6}{x^2-4}$ c) $\lim\limits_{x\to 0}\dfrac{\tan 2x}{\tan 7x}$

d) $\lim\limits_{x\to 0}\dfrac{\sqrt{3+x}-\sqrt{3}}{x}$ e) $\lim\limits_{x\to 3}\dfrac{|x-3|}{x-3}$ f) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

4. Let $g(x)=x^5+x^3+30$. **Without graphing** the function g, use a theorem to show that there is at least one number $c\in(-2,\ 2)$ such that $g(c)=0$.
HINT: don't try to find c !

5. Let

$$f(x)=\begin{cases} x^2+1 & x>2\\ A & x=2\\ 2x+1 & 2>x\geq 0\\ x^2+3 & x<0\end{cases}$$

a)For what value of A is f continuous at $x=2$? **Explain!**
b)Find the following limits or write DNE if the limit doesn't exist. **Show all work.**
$\lim_{x\to 2}f(x)$, $\lim_{x\to 1}f(x)$, $\lim_{x\to 0}f(x)$, $\lim_{x\to(-1)}f(x)$
b) Is $f(x)$ differentiable at $x=0$?

6. Find the following derivatives from the definition:

a) $f(x)=x^2+3x$ b) $g(x)=\dfrac{1}{x+2}$ c) $h(x)=\sqrt{x-3}$

7. The line $y = 2x + 3$ is tangent to the parabola $y = x^2 + B$. Find B.

8. Find the derivative of the following functions. Don't simplify!

a) $f(x) = \dfrac{7}{x^{3/7}} + \sqrt{x^5} + x^7 + 45$ b) $g(x) = (x + 9) * (x^2 - 7x)$

c) $h(x) = \left(\dfrac{x^2 + 7}{x^5 - 8x}\right)^9$ d) $k(x) = \dfrac{(x^4 + 2)^6}{\sqrt[3]{x^3 + 5x}}$ e) $\ln x^5 + x - \ln x$

9. Find the equation of the tangent line for the graph of $f(x) = 2 * \sqrt{x} + x^2 - 5$ at $x = 1$.

10. Let $f(x) = g(\sqrt{x + 3})$. Find $f(6)$ and $f'(6)$.
It is impossible to find g, but it will be useful to use some of the following known values of $g(x)$ and $g'(x)$:
$g(1) = 2$, $g(2) = 5$, $g(3) = 7$, $g(4) = 2$, $g(5) = 11$, $g(6) = 13$ and $g(7) = 21$
$g'(1) = 3$, $g'(2) = 2$, $g'(3) = 8$, $g'(4) = 10$, $g'(5) = 12$, $g'(6) = 21$ and $g(7) = 23$.

11. Sketch a possible graph of F on $[-3, \ 3]$ such that:
F is continuous on $[-3, \ 0)$ and $(0, \ 3]$, $\lim_{x \to 0^+} F(x) = 5$, $\lim_{x \to 0^-} F(x) = -2$, F is not differentiable only at $x = 0$ and $x = 1$.

12. Let $y = 2x^4 + 3x^2 + 12$. Find $\dfrac{d^3y}{dx^3}$.

13. The distance s (in feet) covered by a car t seconds after starting from rest is given by $s(t) = 20t + 6t^2 + t^3$, when $0 \le t \le 20$.
a) What is the velocity of the car 5 seconds after starting from rest?
b) What is the acceleration of the car at that time?

14. Sketch a possible function on the domain $(-2 \ , \ 4)$ that is :
Not differentiable only at $x = (-1.5)$, (-1) , 0 , 0.5 , 1 , 3, not continuous only at $x = (-1)$, 0.5 , 3 and has no limit only at $x = 0.5$, 3.

15. Let $h(x) = f(g(x))$. Assume that $f(1) = 2$, $f'(1) = 7$ $f(2) = 5$ $f'(2) = 5$, $g(1) = 2$ *and* $g'(1) = 3$. Find $h'(1)$, $(fg)'(1)$, $(f/g)(1)$.

16. Suppose that f and g are differentiable functions such that $f(g(x)) = 8x^2$ to all real numbers x. Assume that $f(2) = 7, \quad g(2) = 4, \quad f'(2) = 4 \quad and \quad f'(4) = 2$. What is $g'(2)$?

17. Give an exact solution for the following population growth problem: The number of students taking 135 increased from 2,300 to 3,200 in the last 2 years. Assuming that this is an exponential growth, find K, the growth constant and use it to find the number of 135 students 3 years from now.

18. Expand the following expression: $\ln \dfrac{\sqrt[5]{x}x^3y^2}{6\sqrt{y}x^9}$