Instructions for Use of Maple in Mathematics 251

Fall 2003

Getting started. The computer program **Maple** is a powerful tool which can help you solve a wide range of mathematical problems: it can differentiate, integrate, and otherwise manipulate mathematical formulas, perform arithmetic calculations, plot curves and surfaces in two and three dimensions, solve differential equations, and carry out a variety of other useful mathematical operations. At Rutgers, Maple is available on the machines in the student computer labs and on the student computer **eden**. The current version of Maple is **Maple9**, which was released in the summer of 2003. It is scheduled to be installed throughout the student labs by the beginning of the Fall 2003 semester. Descriptions of projects will be given for Maple9, but the projects have been tested on several versions, so should run on whatever version is available. On all systems, there is a **text based** version and **graphical worksheet interface** versions. Only the graphical version will be used here. On Unix systems, like eden, this graphical version is called **xmaple**, since it is customized to run under the X windows system. Since there are no longer X-terminals in the student labs, most work will be done on PCs., where all versions of Maple are usually installed on the "Course Related Software" section of the *Start Menu*. It is also expected that the Microsoft Windows desktop on the lab PCs will be taught to recognize Maple worksheets so that you can start Maple by selecting the file that you want to open. The commands you use to do mathematics with Maple will be the same on all these machines, there are differences among computers, such as in the way that files are opened, saved, and printed.

When you identify yourself to the PC to start a session, be sure to check the box that requests access to your home directory on eden. This will start a program called *WebDrive* that allows you access to files on eden. You will use this to save worksheets between maple sessions.

Starting with Maple 9, each worksheet opens in its own window with a *Menu Bar*, consisting of a row of *menu buttons* (**File, Edit** etc.) At the top of the window. Underneath the row of menu buttons is the *Tool Bar*, most of which are shortcuts to menu commands. Directly below the *Tool Bar* is the *Context Bar*, which consists of more buttons which are shortcuts to menu commands. If you have enabled **Tool Tips** from the **Interface** page of the Options menu, when you place the mouse pointer on any of these buttons, a rough idea of what the button does will appear. The rest of the window, except far a small *Status Bar* at the bottom is devoted to the worksheet. You can now give an instruction to Maple by placing the mouse pointer on the worksheet (just to the right of the prompt >), typing a Maple command, and then typing a carriage return. Maple carries out the command and prints a response. For example, if you type `1 + 1;` , Maple will respond with **2**. Note that each Maple command ends with a semicolon. (If you forget the semicolon, Maple will remind you.) If you select part of the worksheet, the right mouse button brings up a **Context menu** that can be useful in editing the worksheet.

Getting Help. Since you will not be given a Maple manual, you should learn to use Maple by using the built in **Help** facility. There is a **Help Menu** at the right end of the menu bar. Selecting any of the items in this menu will open the **Help Browser Window** (introduced in Maple 9 — a less convenient set of help window were used in earlier versions). New users are presented with this **Help Browser Window** when Maple starts. Only one Help page at a time will be visible, but the browser includes a **History** panel as well as an index and a search tool.

Maple Worksheets. The window which appears when you start Maple is called a **Maple worksheet.** To complete each computer assignment in Mathematics 251 you are asked to turn in an **edited** printout of your work; such a printout can be obtained by editing, and then printing, your worksheet. The printout should include only the numerical, symbolic, and graphical output of Maple which is appropriate for the solution of the problems assigned, plus text material interpreting the results obtained by Maple. Using the **T** tool will insert a new line into the worksheet in text mode, allowing descriptive material to be added. (**Undocumented Feature**: Prior to Maple 9, this tool modified the current line, but this change is not described in the documentation. There is a place in the *Insert* menu for inserting text, but this does not change the nature of the line where it is used, so use of the tool is preferred.) This can also be used to label graphical output, but better results can be obtained using the **title option** in the plot instruction. Maple includes various editing capabilities which should enable you to produce neat and coherent output, and which we now describe.

To remove an unwanted portion of your Maple worksheet (e.g., a region containing commands that you typed incorrectly or that were not directly relevant to the solution of the exercises), select the region to be deleted by clicking the left mouse button at the beginning, then dragging the mouse across to the end of the portion of the worksheet you wish to delete. The region should now be highlighted. The **Delete** key will purge what you have selected. A safer approach is to choose **Cut** from the **Edit** menu or toolbar. This allows you to copy the removed region to a new location, select the region as above by selecting **Paste** from the **Edit** menu or toolbar. There is also a **Copy** item in the **Edit** menu. that makes a region available for pasting without removing it from the worksheet. These operations are borrowed from your computer's *Window Manager*, so it should be possible to copy parts of your Maple worksheet to other applications. However, this may not work as smoothly as it does with applications that are more closely integrated with the system. You can delete whole lines with the *Ctrl-Delete* key combination. You should experiment with the different aspects of the user interface (mouse, keyboard, menu or toolbar items) to find the method that you find easiest for customizing the appearance of your worksheet.

Sometimes it is useful to be able to place a comment after a Maple command, rather than insert text elsewhere in the worksheet. To do this, enter the sharp symbol #. Everything typed on a line following this symbol will be considered by Maple to be a comment, and therefore not executed.

To make your worksheet less cluttered, it is a good idea to have Maple suppress the output of various commands, e.g., the command `with(plots)` or a command given to assign a name to a plot. To do this, end the command with a colon (:), instead of a semicolon (;).

Obtaining Copies of the Labs in Worksheet Form. In some of the labs, part of the lab will ask you to execute a string of Maple commands to learn what they do. To avoid retyping these commands, you can first obtain a modified copy of the lab in worksheet form (a "seed file") from the course web page. This modified copy will omit instructions and problems and contain only strings of some of the Maple commands you are asked to execute. Once you obtain this file, you can access it by following the instructions in the next section. To save typing, outlines of the commands described in the labs have been saved in "seed files". These can be obtained from the course page using your internet browser. When you have saved the file in your home directory, you can load it into a Maple session from the **File:Open** menu.

Opening an Existing Worksheet. To open an existing worksheet, choose **Open** from the **File** Menu or the appropriate tool. Navigate to the correct directory and select the name of the file you wish to open and then click on **OK**.

Note: Although you will see the output from your previous work, **none** of the definitions made there are currently active. You will need to select **Execute Worksheet** from the Edit menu or toolbar. You may also

hit the **Enter** key on individual lines to step through the worksheet, editing it as you restore your previous state. By contrast, **everything** you did in the current session on different worksheets is remembered. The `restart;` instruction can be used to cause Maple to clear previous definitions and revert to its initial state. Depending on configuration, material in different worksheets may be either shared or kept separate. Until you have more experience, it is better to work with only one worksheet at a time, begin your worksheet with a `restart;` instruction, and use **Execute Worksheet** after making many changes to restore the connection between the order of the input and output in the worksheet.

To continue, you will need to know some basic commands and syntax of Maple.

Arithmetic. The operations addition, subtraction, multiplication, division, are indicated by `+, -, *,` respectively. Fractions are represented exactly, and Maple automatically simplified them. Thus, if you type `1/3+1/6;` , Maple will answer $1/2$. Exponentiation is also available with the operator `/, ^`, but it should be used sparingly. Many expressions that we write using exponents are better described using the **exponential function** described in a later section.

When operating on integers or fractions, Maple does exact arithmetic, rather than using decimal approximations. To get a decimal approximation, use the Maple command `evalf`. The Maple command `evalf(4/7);` produces a 10 digit approximation to $4/7$—as will typing `4.0/7.0;` . Additional accuracy can be obtained by including a second argument: typing `evalf[20](4/7);` will produce a 20 digit approximation. (Note: this form of the `evalf` function is now standard, replacing an older form `evalf(4/7,20);`. The older form is still accepted, but no longer described in the *Help* file. You can change also precision for your whole Maple session by setting a new value to the reserved variable `Digits`.

Algebra. All grouping of expressions is done with the left and right parentheses — (and). Although the usual rules of precedence apply, parentheses can always be used to clarify your intent. Variables are not restricted to single letter names, as in Elementary Algebra (and most Calculus textbooks). You can use (almost) any string of letters and numbers that starts with a letter. As a result, the product xy **must** be written `x*y`, not `xy` or `x y`; if you type `xy`, Maple assumes you are referring to a variable called "`xy`". Thus, to enter the expression $(2x + y^2)/(x^2 + y) + 1$ into Maple, you type:

```
(2*x +y^2)/(x^2 + 2*y) + 1;
```

Recent versions of Maple allow you to use a **palette** (available from the **View** menu) to help build expressions. Projects for this course use a "seed file" containing formulas given in the description with space for you to add new formulas as you work through the project. You can add more space using the **Execution Group** item of the **Insert** menu or the button on the toolbar that looks like the [> Maple prompt.

To help you do algebraic manipulations, Maple has the commands `expand`, `factor`, and `simplify`, which you can learn about by using the **Help** facility. It is often necessary to `expand` and expression before Maple will know what it means to `simplify` it. In particular, Maple considers

```
(2*x +y)/(x + 2*y);
```

to be simple.

Algebraic numbers like the square root of 2 are considered exact quantities. If you enter

```
a:=(sqrt(2)+1)^10;
```

Maple will appear to simply echo a *prettyprint* version of that statement. To get an answer in the form 3363+2378*sqrt(2), you need to follow this with `expand(a);` .

You can also solve algebraic equations by using the commands `solve` (for exact answers) or `fsolve` (for numerical answers to the accuracy specified by `Digits`).

The standard functions. Note that the exponential function is built into Maple and is referred to as `exp`. The number *e*, if you ever need it, is `exp(1)`. If you have the fragment `e^x` in a worksheet, it will *look* right, but will usually give incorrect results when you try to *use* it. Similarly, the number π has the special name `Pi` — **not** `pi` which looks the same, but will be treated as an ordinary variable. If you want to, you can say `pi:=3;` but this won't affect the value of π or anything that depends on it. However, Maple will fuss at you if you try to change the value of `Pi`. Maple also recognizes names like `log`, `ln`, `sin`, `cos`, `tan` as standard functions. Those names are also protected against use as names of variables. Normally, `log` is treated as a synonym for `ln` — to get the *common logarithm* of 2 that your calculator uses, you should write `log10(2.);` or `log[10](2.);`. The Help page, "Initially known mathematical functions" will give you a list of names of the functions that are always avaialable.

User-defined Functions and Expressions. In Maple, `x^2 - 2*x +3` is an **expression**. You can assign a name to this expression for future use by typing `g:= x^2 - 2*x +3;`. Note that a colon is required before the equal sign in an assignment statement. Expressions can contain several variables, as in `h:= y*t - sin(y)`. To evaluate an expression at a particular value, use the Maple command `subs`. For example, `subs(x=2,g);` will produce the value 3. Maple also has a construct called a **function**, defined by statements like `f:=x -> x^2 - 2*x +3;`.

We won't often use functions, since it is easier to work with expressions, but you might see them in examples on **Help** screens. Some *seed files* may also define functions to extend the Maple language on that worksheet. While we don't expect you to *write* functions in the Maple programming language, you should have no difficulty *reading* these definitions.

Plots. The basic plotting command in Maple is `plot`. This command has many forms—for example, several functions can be plotted at once—so you should look carefully at the examples given at the end of its **Help** page to get some idea of its flexibility. There are many other plotting commands in Maple; in this course we will use primarily `plot3d` and `implicitplot`. To use some of the plotting commands you must first issue the command `with(plots);`. For example, this is necessary in order to use the command `implicitplot`. It is frequently useful to enlarge the size of a plot. To do this, place the mouse pointer in the region occupied by the plot and click the left mouse button. This places a box around the plot and changes the Menu Bar and Control Bar (and context menu) so that new options are offered to manipulate the plot. The plot is resized by placing the mouse pointer at one of the small dots along the edge of the box and dragging the pointer. When you have finished modifying the plot, move the mouse down to the next prompt $>$ and click the left mouse button. When viewing three dimensional plots, it is useful to view the plot from different viewpoints. First, place a box around the plot by moving the mouse pointer in the region occupied by the plot and clicking the left mouse button. Now, place the mouse pointer inside the box and while holding down the left mouse button, move the mouse pointer to different positions. Explore the effects of using the **Axes** and other commands on the **Menu** Bar. (In the current version, you see changes immediately, but earlier versions required you to select **Redraw** from a menu in order to see your changes.)

Vectors. As of Maple6, Maple now has two different packages for linear algebra. These packages allow powerful operations to be introduced simple by calling them by name, but they impose a complicated structure on everything they touch. As a result, simple things become more difficult. In this course, we are mostly concerned with vectors as triples of numbers to represent points in space, and the operations

that we need have been described simple using that notation. The generality used in Linear Algebra is not needed. Maple includes a structure called a **list** that is easy to use and has all the properties that we need. You describe a list by enclosing a sequence of quantities separated by commas in square brackets. Thus, `[1, 3, -1]` describes a point, and `[cos(t),sin(t),t]` describes a *helix*. The description of a space curve like a helix can be directly differentiated with respect to its parameter as a step in using Calculus to find properties of the curve. However, a special construction is needed to multiply all elements of a list by a single expression. This is one place where a function is used to extend the Maple language.

Differentiation and Integration. In addition to performing basic calculations and evaluating expressions containing standard functions, Maple can also differentiate and integrate. The `diff` command differentiates expressions. For example, `diff(x*sin(x), x);` differentiates $x\sin(x)$ with respect to x. If you have named an expression, it is a useful convention to introduce a modification of that name as a name for the derivative (or partial derivatives). That is, if you have `y:=x*sin(x);`, you can write `y1:=diff(y,x);`. A second derivative can then be found, and named, quickly by writing `y2:=diff(y1,x);` to differentiate the first derivative using the name that you introduced. If you need a second derivative without seeing the first derivative, either `diff(x*sin(x), x, x);` or `diff(x*sin(x),x$2);` may be used. If you have typed `g:=x^2*y;` , then `diff(g,x);` is the first partial derivative of x^2y with respect to x and `diff(g,x,y);` is the mixed second partial derivative.

Note that Maple assumes that everything not involving the variable with which it is differentiating is constant. This means that no special notation is required to distinguish partial derivatives of expressions from ordinary derivatives. Although we claim to be differentiating **functions** in Calculus, the techniques are based on examining how **variables** appear in **expressions**, so the way that Maple works with expressions is more familiar.

The command for both definite and indefinite integration is `int`. If Maple cannot evaluate a definite integral exactly, numerical integration may be used. Definite integrals are evaluated using a single instruction written like `int( sin(x), x=0..Pi );` to get

$$\int_0^{\pi} \sin x \, dx.$$

It is not necessary to apply the `subs` instruction to the result of indefinite integration, although that approach gives the same answer. The role of dx in the mathematical notation for integrals is played by including the name of the variable x in the second argument of the `int` instruction.

Printing your Maple Worksheet. To print your Maple worksheet, choose **Print** from the **File** menu or click on the print button in the **Tool Bar** (its position varies, but it is with the other file tools at the left and it looks like a printer). A panel will appear with the options "Output to File" and "Print Command" at the top. Clicking on "Print Command" will direct your output to the printer if the command has been correctly entered, while clicking on "Output to File" places your output in a file, whose name will be chosen in another box (the default is *out.ps*). If you wish to change the file name or the print command, click in the box and type your changes. Since the file will be a postscript file, the file name should have the form *something.ps*. When you have finished making your choices, click on the "Print" button.

There is also a **Print Preview** that lets you see how your worksheet will appear before you commit it to paper.

There is a item in the *Insert* Menu that allows you to add a page break to your worksheet. In Maple 9 the location of the break is shown in the worksheet on your screen.

Saving your Maple Worksheet. If your work is interrupted, you can save your work so that you can later resume where you left off. To save your Maple worksheet, choose **Save As** from the **File** menu. Then type the name of the file in which you wish to save your worksheet in the **Selection** box where the mouse pointer is (replacing the asterisk). The file name should have the form *something.mws* for compatibility with earlier versions of Maple, or *something.mw* for the xml format introduced with Maple 9. After you have typed the filename, click on "OK" to save the file. Once you have named the worksheet and saved it in a file, you can save further changes by choosing **Save** from the **File** menu. Maple automatically keeps track of the filename.

Ending your Maple Session. To end your Maple session, choose **Exit** from the **File** menu. When you close a worksheet, or exit the program, Maple will prompt you to save worksheets that have changed since the last save.

Useful Commands and Techniques. If Maple gets hung up in a calculation or is taking too long, click the mouse on the **stop** button in the **Tool Bar** menu.

Any Maple command previously entered in your worksheet can be re-executed without retyping it in a new location. Simply move the mouse to the position of the command you wish to execute and hit the **Enter** (or **Return**) key.

It is often useful to be able to refer later to the result of a computation—the output of some command—in a simple way. To make this possible, simply assign the output of the command to a variable. For example, if you enter `a:= evalf(2*Pi);` then you can later square the result of `evalf(2*Pi);` by typing `a*a;`. It you find that you need a recent result that was not named, the special names `%, %%`, etc. may be used to refer to the previous result, the one before that, etc. (the instructions are not clear about how much of the history can be retrieved in this way). A better solution might be to add a name the line that computed the expression and re-execute that line.

You can assign a name to a plot just as described above for assigning a name to an expression. Several previously named plots can then be displayed on the same graph by using the command `display`. This is part of the `plots` library. Looking in the documentation for that library may be the easiest way to find the help page for this function.

Only the most recent assignment to a name is remembered. You can unassign `a` by typing `a:='a';`. To clear all the assigned variables in a Maple session, you can type `restart;` as indicated earlier. If you are unsure of what value has been assigned to the name `a`, you can type `a;` at the current prompt. Such working notes should be edited out of the worksheet before you submit it.

Getting Help From Other Students.

The purpose of the Maple assignments is partly to learn about Maple, a very useful program for symbolic, numerical, and graphical computations, and partly to help you understand the material in the course. Just as with other homework assignments, it is permissible and helpful to discuss the Maple labs with other students. However, the Maple labs you are turning in are being graded and will be part of your final course grade, and the printed form of the worksheet is expected to be the work of the student who submits it. The grader will concentrate on the text comments that interpret the results of the computation, and these are expected to reflect your **individual understanding** of the topic.