**Name** ______________________________ **MA251 Final Exam Dec. 16, 1998**

1. (10 points) Given $P(1,2,3)$, $Q(-1,1,1)$, $R(2,0,2)$, $S(5,-1,-2)$, find the volume of the parallelpiped with adjacent sides $PQ$, $PR$, and $PS$. (ANSWER: 25)

2. (10 points) Find the equation of the normal plane to the curve $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$ at the point where $t = 2$. (ANSWER: x + 4y +12z = 114 )

3. (12 points) Find the length of the curve $\mathbf{r}(t) = \cos(2t)\,\mathbf{i} + \sin(2t)\,\mathbf{j} + t\,\mathbf{k}$, $0 \le t \le 2\pi$. (ANSWER: $2\sqrt{5}\pi$ )

4. (14 points) Find the tangential component $a_T\mathbf{T}$ of acceleration if $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$ and $t = 1$. (ANSWER: $\langle \frac{11}{7}, \frac{22}{7}, \frac{33}{7} \rangle$ )

5. (10 points) Find the position vector $\mathbf{r}(t)$ if $\mathbf{a}(t) = t^2\,\mathbf{i} + t\,\mathbf{j} + e^t\,\mathbf{k}$, $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$, and $\mathbf{r}(0) = \mathbf{k}$. (ANSWER: $\mathbf{r}(t) = (\frac{t^4}{12} + t)\mathbf{i} + (\frac{t^3}{6} + t)\mathbf{j} + (e^t - t)\mathbf{k}$ )

6. (12 points) Find $\dfrac{\partial z}{\partial x}$ if $x^3 + y^3 + z^3 = xy + xz + yz$. (ANSWER: $\frac{3x^2-y-z}{-3z^2+x+y}$ )

7. (12 points) If $z = xy\sec(x)$, $x = t^2 + u$, and $y = u^2 + tv$, find $\dfrac{\partial z}{\partial u}$ when $t = 1$, $u = -1$, and $v = 3$. (For those who've forgotten, $\frac{d}{dx}\sec x = \sec x \tan x$). (ANSWER: 4 )

8. (15 points) Find all local maxima, minima, and saddles of the function $f(x,y,z) = x^3 - 6xy + 8y^3$. (ANSWER: saddle at (0,0) and local min at $(1,\frac{1}{2})$ )

9. (12 points) Calculate the integral $\int_0^1 \int_x^1 e^{x/y}\,dy\,dx$ by first reversing the order of integration. (ANSWER: $\frac{e-1}{2}$ )

10. (15 points) Find the maximum and minimum values of $f(x,y) = x^2y$ subject to the constraint $x^2 + y^2 = 1$. (ANSWER: max = $\frac{2\sqrt{3}}{9}$, min = $-\frac{2\sqrt{3}}{9}$ )

11. (12 points) Find the volume above the paraboloid $z = x^2 + y^2$ and below the half cone $z = \sqrt{x^2 + y^2}$. (ANSWER: $\frac{\pi}{6}$ )

12. (12 points) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \sin x\,\mathbf{i} + \cos y\,\mathbf{j} + xz\,\mathbf{k}$ and $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^4\,\mathbf{k}$, $0 \le t \le 1$. (ANSWER: $\frac{13}{9} - \cos(1) + \sin(1)$ )

13. (12 points) If $\mathbf{F} = 4xe^z\,\mathbf{i} + \cos y\,\mathbf{j} + 2x^2e^z\,\mathbf{k}$, find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$. (ANSWER: $f(x, y, z) = 2x^2e^z + \sin(y)$ )

14. (15 points) Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = x^2y\,\mathbf{i} - 3y^2\,\mathbf{j}$ and $C$ is the curve $x^2 + y^2 = 1$. (ANSWER: $-\frac{\pi}{4}$ )

15. (15 points) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = 2z\,\mathbf{i} + 4x\,\mathbf{j} + 5y\,\mathbf{k}$ and $C$ is the curve of intersection of the plane $z = x + 4$ and the cylinder $x^2 + y^2 = 4$. (ANSWER: WE SKIPPED STOKES' THEOREM. )

16. (12 points) Use the Divergence Theorem to calculate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = x^3\,\mathbf{i} + y^3\,\mathbf{j} + z^3\,\mathbf{k}$ and $S$ is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$. (ANSWER: $11\pi$ )

**Bonus.** (10 points) Find the simple closed curve for which the line integral $\int_C (y^3 - y)dx - 2x^3dy$ is a maximum. (ANSWER: $C$ is given by $6x^2 + 3y^2 = 1$, but you had to supply a valid reason to get credit. )