1. (12 points) If $\mathbf{a} = \langle 1, 2, 3 \rangle$ and $\mathbf{b} = \langle 3, -1, 3 \rangle$, find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$.

2. (12 points) Evaluate the integral $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx$ by reversing the order of integration.

3. (12 points) Find the equations of the tangent plane and normal line to the surface described by $z + 1 = xe^y \cos(z)$ at the point $(1, 0, 0)$.

4. (14 points) If $e^{xy} - e^{zx^2} = 1$, find $\dfrac{\partial z}{\partial x}$.

5. (12 points) Find the local maxima, minima, and saddles of $f(x, y) = x^3y + 12x^2 - 8y$.

6. (12 points) Find the absolute maximum and minimum of $f(x, y) = x^2 - y^3$ subject to the constraint $2x^2 + 3y^2 = 5$. Don't worry if you don't have a calculator. You'll get credit if you find all of the potential maxima and minima.

7. (14 points) Find a function $f(x, y, z)$ so that $\nabla f = \langle zye^{xy},\, zxe^{xy} + x,\, e^{xy} + 2z \rangle$ and use it to evaluate the line integral $\int_C \nabla f \cdot d\mathbf{r}$ along any curve from $(1, 0, 0)$ to $(0, 1, 1)$.

8. (12 points) Find the volume under the cone $z = \sqrt{x^2 + y^2}$ which lies above the ring bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ in the $xy-$plane.

9. (14 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x^2\,\mathbf{i} + xy\,\mathbf{j} + z^2\,\mathbf{k}$ and $\mathbf{r}(t) = \sin(t)\,\mathbf{i} + \cos(t)\,\mathbf{j} + t^2\,\mathbf{k},\ 0 \le t \le \pi/2$.

10. (12 points) For what value of x is the curvature of $y = x^3$ a maximum?

11. (12 points) Find $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and S is the sphere $x^2 + y^2 + z^2 = 9$.

12. (12 points) Evaluate $\oint_C (3y - e^{\sin x})\, dx + (7x + \sqrt{y^4 + 1})\, dy$, where C is the boundary of the rectangle $0 \le x \le 2,\ 0 \le y \le 3$.

13. (12 points) Find $\int\int_S z\, dS$, where S is the surface with parametric equations $x = \cos(u),\ y = \sin(u),\ z = v,\ 0 \leq u \leq 2\pi,\ 0 \leq v \leq 2$.

14. (12 points) Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$.

Bonus Problem (10 points) Let S and E be as in the statement of the Divergence Theorem. Prove that

$$\int\int_S (f\nabla g) \cdot d\mathbf{S} = \int\int\int_E (f\nabla^2 g + \nabla f \cdot \nabla g)\, dV.$$