PRACTICE PROBLEMS—EXAM 2

Please note that this is *not* a practice exam; in particular, there are more problems here than will be on the exam. Moreover, although these problems are generally similar to exam problems, it is possible that the exam will contain some problems quite different from any here.

The answers are not guaranteed.

1. Use Lagrange multipliers to find the maximum and minimum values of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$. 30 at (1,−2,5); −30 at (−1,2,−5)

2. Let D be the triangular region bounded by the lines $x = 0$, $y = 0$, and $2x + 3y = 1$. Sketch the region D and evaluate $\int\int_D x\,dA$. 1/72

3. Sketch the region of integration and write an equivalent integral with the order of integration reversed for the integral $\displaystyle\int_0^2 \int_{x^2}^{2x} xy\,dy\,dx$. Evaluate the integral in both forms. $\int_0^4 \int_{y/2}^{\sqrt{y}} xy\,dx\,dy = 8/3$

4. Change the Cartesian integral $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-(x^2+y^2)}\,dy\,dx$ into an equivalent polar integral and evaluate the polar integral. $\int_0^{\pi/2} \int_0^2 e^{-r^2} r\,dr\,d\theta = \pi(1-e^{-4})/4$

5. Find the area of one leaf of the "'rose" $r = \sin 3\theta$. $\pi/12$

6. Find the area of that portion of the "saddle" $z = 3x^2 - 3y^2$ which lies inside the cylinder $x^2 + y^2 = 4$. $\pi[(145)^{3/2}-1]/54$

7. Express the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + 2y + z = 4$ as an iterated triple integral and then evaluate the integral. 16/3

8. Let E denote the region in the first octant that is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$. Express the volume of E as in iterated triple integral in (i) cylindrical and (ii) spherical coordinates. Then evaluate both integrals. $9\pi(2-\sqrt{2})/4$

9. Evaluate the integral $\int\int_D xy\,dA$ by the transformation $x = u/v$, $y = v$. Here D is the region in the first quadrant bounded by the curves $y = x$, $y = 3x$, $xy = 1$, and $xy = 3$. $2\ln 3$

10. Find the gradient vector field of the function $f(x,y) = xy$, and sketch this field in the region $-2 \le x \le 2$, $-2 \le y \le 2$. $y\mathbf{i}+x\mathbf{j}$

11. Evaluate the line integral $\int_C f\,ds$ of $f(x,y,z) = x - y + z - 2$ along the curve C given by $\mathbf{r}(t) = t\mathbf{i} + (1-t)\mathbf{j} + \mathbf{k}$ for $0 \le t \le 1$. $-\sqrt{2}$

12. Find the work $\int_C \mathbf{F}\cdot d\mathbf{r}$ done by the force field $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j}$ on a particle that moves from $(1,0)$ to $(-1,0)$ (a) along the curve given parametrically by $\mathbf{r}(t) = \langle 1-2t, t(1-t)\rangle$, $0 \le t \le 1$, (b) along the parabola $y = x^2 - 1$. −1/15, −16/15

13. Determine whether or not the vector field $\mathbf{F}(x,y,z) = y\mathbf{i} + x\mathbf{j} + \mathbf{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$. $f(x,y,z)=xy+z+K$

14. Use Green's Theorem to evaluate the line integral $\oint_C (2xy^3\,dx + 4x^2y^2\,dy)$ along the curve C: boundary of the 1st quadrant region enclosed by the curves $y = 0$, $x = 1$ and $y = x^3$. 2/33

15. If $\mathbf{F}(x,y,z)$ and $\mathbf{G}(x,y,z)$ are vector fields, show that $\operatorname{div}(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\operatorname{curl}\mathbf{F} - \mathbf{F}\cdot\operatorname{curl}\mathbf{G}$.