Please note that this is *not* a practice exam; in particular, there are more problems here than will be on the exam. Moreover, this set of problems does not provide exhaustive coverage of all material in the course; you should study also your earlier exams and the corresponding review problems. Finally, although these problems are generally similar to exam problems, it is possible that the exam will contain some problems quite different from any here. The answers are not guaranteed.

1. A particle moving in the plane has coordinates $x(t) = 3\cos(t^2+1)$, $y(t) = 3\sin(t^2+1)$ for $t > 0$.
(a) Find the position vector, velocity, speed, and acceleration of the particle at time t.
$\langle 3\cos(t^2{+}1), 3\sin(t^2{+}1)\rangle$; $\langle -6t\sin(t^2{+}1), 6t\cos(t^2{+}1)\rangle$; $6t$; $\langle -6\sin(t^2{+}1) - 12t^2\cos(t^2{+}1), 6\cos(t^2{+}1) - 12t^2\sin(t^2{+}1)\rangle$
(b) Find $\mathbf{T}$, $\mathbf{N}$, and κ at time t. $\langle -\sin(t^2{+}1), \cos(t^2{+}1)\rangle$; $\langle -\cos(t^2{+}1), -\sin(t^2{+}1)\rangle$; $1/3$
(c) Find the tangential and normal acceleration vectors. $6\mathbf{T}$; $12t^2\mathbf{N}$
(d) What geometric path does the particle follow? A circle, radius 3, center at origin.

2. A particle moving in space has acceleration at time t given by $\mathbf{a}(t) = 2\mathbf{i} - \mathbf{j} + 3t\mathbf{k}$. The particle passes through the origin at time $t = 0$ with velocity $-\mathbf{i} + 2\mathbf{j}$. Find its position at time t and its velocity as it passes through the point $(2, 2, 4)$. $\mathbf{r} = \langle t(t-1), t(2-t/2), t^3/2\rangle$; $\mathbf{v} = \langle 3, 0, 6\rangle$

3. Let $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$.
(a) Find $\mathbf{a}\cdot\mathbf{b}$ and $\mathbf{a}\times\mathbf{b}$. 7; $\langle -5, 3, 4\rangle$
(b) Find the equation of a plane parallel to both $\mathbf{a}$ and $\mathbf{b}$ and passing through the point (2,1,-1).
(c) Find the distance from the point $(3, 2, -5)$ to the plane of (b). $5x - 3y - 4z = 11$; $18/\sqrt{50}$

4. Let $f(x, y, z) = xy + yz + zx$, and let P be the point $P(1, 2, 3)$.
(a) Find the direction in which the function is increasing the most rapidly at the point P, and the corresponding rate of increase. $\langle 5, 4, 3\rangle/\sqrt{50}$; $\sqrt{50}$
(b) Find one direction in which the directional derivative of the function at P is 0. $(-4\mathbf{i} + 5\mathbf{j})/\sqrt{41}$
(c) Find the directional derivative of f in the direction of $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j} - 10\mathbf{k}$ at P. $-2/\sqrt{30}$
(d) Find the equations of the tangent plane and normal line to the surface $f(x, y, z) = 11$ at P.
(e) Use differentials to estimate $f(1.1, 1.8, 3.2)$. *(d)* $5x + 4y + 3z = 22$; $x = 1 + 5t$, $y = 2 + 4t$, $z = 3 + 3t$; *(e)* 11.3

5. Find the absolute maximum and minimum values of $f(x, y) = x^2 - 6x + y^2 - 8y + 5$ in the region $x^2 + y^2 \le 36$. (Hint: you must look for critical points and also check for extreme values on the boundary.) Min: -20 at $(3, 4)$, Max: 101 at $(-18/5, -24/5)$

6. Suppose that $w = x^2y - y^3$ with $x = f(r, s)$, $y = g(r, s)$. Find $\left.\dfrac{\partial w}{\partial r}\right|_{\substack{r=2\\s=1}}$ and $\left.\dfrac{\partial^2 w}{\partial r\partial s}\right|_{\substack{r=2\\s=1}}$, given

$$f(2,1) = 1, \quad g(2,1) = -2 \quad f_r(2,1) = 2, \quad f_s(2,1) = -1 \quad g_r(2,1) = 3, \quad g_s(2,1) = -2$$
$$f_{rr}(2,1) = 4, \quad f_{rs}(2,1) = 2 \quad f_{ss}(2,1) = 2 \quad g_{rr}(2,1) = -3, \quad g_{rs}(2,1) = 0, \quad g_{ss}(2,1) = 4.$$

-41; -86

7. Suppose that $f(u, v, w)$ is a function with continuous partial derivatives and that $g(x, y, z) = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$. Show that $\dfrac{1}{x}\dfrac{\partial g}{\partial x} + \dfrac{1}{y}\dfrac{\partial g}{\partial y} + \dfrac{1}{z}\dfrac{\partial g}{\partial z} = 0$ for x, y, and z nonzero.

8. Compute $\iiint_D (x^2 + y^2)\, dV$, where D is the region inside the hemisphere $x^2 + y^2 + z^2 = 9$, $z \ge 0$, and between the cones $z^2 = x^2 + y^2$ and $z^2 = 3(x^2 + y^2)$. Be sure to use the easiest coordinate system. $9\pi(\sqrt{3} - \sqrt{2})$; $81/20(9\sqrt{3} - 10\sqrt{2})\pi$

9. Find the surface area of the portion of the surface $z - x^2 - y^2 = 9$ which lies between the planes $z = 10$ and $z = 13$. $\pi(17^{3/2} - 5^{3/2})/6$

10. For the integral $\int_0^4 \int_{x^2-4x}^{x} (x+2y)\,dy\,dx$: sketch the region of integration, evaluate the integral, and write down the integral with the reversed order of integration.

448/15; $\int_{-4}^{0}\int_{2-\sqrt{y+4}}^{2+\sqrt{y+4}}(x+2y)\,dx\,dy+\int_0^4\int_y^4(x+2y)\,dx\,dy$

11. Let R be the region for which $(x-4)^2+y^2 \le 16$ and $x \ge y$. Evaluate $\iint_R xy\,dA$ using polar coordinates. $-64/3$

12. **Perverse problems:** it should be easy to check your answers!
(a) Find the area of the rectangle $0 \le x \le \sqrt{3}$, $0 \le y \le 1$ using an integral in polar coordinates.
(b) Find the area between the circles $r = 2\cos\theta$ and $r = 6\cos\theta$ using an integral in polar coordinates.
(c) Find the volume inside the sphere $x^2+y^2+z^2 = 9$ and outside the sphere $x^2+y^2+z^2 = 1$ using cylindrical coordinates.

13. Let C be the triangle in the xy plane with vertices at (0,0), (1,0), and (0,1), oriented counterclockwise.
(a) Evaluate $\oint_C 2y^2\,dx + 2x\,dy$ directly as a line integral. 1/3
(b) Evaluate the line integral in (a) by using Green's Theorem and then evaluating a double integral.

14. Do Exercise 31 on page 910 and Review Exercises 1–8 on page 943 of Stewart. 2.T, 4.T, 6.T, 8.F

15. Use the curl operator to show that the vector field $\mathbf{F} = \langle 3x^2y^2z+1, 2x^3yz+2, x^3y^2+3\rangle$ is conservative, and find f with $\mathbf{F} = \nabla f$. $x^3y^2z+x+2y+3z$

16. Let E be the solid region $3 \le z \le 4-(x^2+y^2)$, S its boundary surface, and $\mathbf{F}$ the vector field $\mathbf{F} = (y+x)\mathbf{i} + (y-x)\mathbf{j}$. Verify the divergence theorem $\iiint_E \operatorname{div}\mathbf{F}\,dV = \iint_S \mathbf{F}\cdot d\mathbf{S}$ for $\mathbf{F}$ and E by calculating both sides and showing that they are equal. $\pi=\pi$

17. Let $\mathbf{a}(x,y,z)$ be the vector field $\mathbf{a} = (x+y)\mathbf{i} + (y-x)\mathbf{j} + z\mathbf{k}$.
(a) Evaluate $\int_\Gamma \mathbf{a}\cdot d\mathbf{r}$ for Γ the straight line segment joining (0,1,2) to (-1,2,0). -1
(b) Evaluate $\oint_C \mathbf{a}\cdot d\mathbf{r}$ for C the circle $x^2+y^2 = 4$, oriented *clockwise*, in the plane $z = 0$. 8π
(c) Let S be the hemisphere $x^2+y^2+z^2 = 4$, $z \le 0$. Using (b), check that Stokes' Theorem $\int_C \mathbf{a}\cdot d\mathbf{r} = \iint_S \operatorname{curl}\mathbf{a}\cdot d\mathbf{S}$ is satisfied here. $8\pi=8\pi$

18. Let S_1 be the disk $x^2+y^2 \le 1$ in the plane $z = 0$, let S_2 be the hemisphere $x^2+y^2+z^2 = 1$, $z \ge 0$, and let $\mathbf{F}$ be a twice differentiable vector field defined in all of space.
(a) Show that Stokes' Theorem implies that

$$\iint_{S_1} \operatorname{curl}\mathbf{F}\cdot\mathbf{n}\,dS = \iint_{S_2} \operatorname{curl}\mathbf{F}\cdot\mathbf{n}\,d\sigma, \qquad (**)$$

where the unit normal vectors $\mathbf{n}$ are chosen to have positive z component ($\mathbf{n}\cdot\mathbf{k} > 0$).
(b) Show that $\operatorname{div}\operatorname{curl}\mathbf{F} = 0$.
(c) Derive the equality (**) from the Divergence Theorem.