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★**Симметрийный анализ и точные решения нелинейных уравнений математической физики. (Russian) [Symmetry analysis and exact solutions of nonlinear equations of mathematical physics]**

‘‘*Naukova Dumka*’’, *Kiev*, 1989. 336 *pp*. 4.50 r. *ISBN* 5-12-000544-6

This book presents the modern development of the (Lie) group-theoretic analysis of linear and nonlinear multidimensional equations of mathematical physics which are invariant with respect to Poincaré, Galilei, conformal, Schrödinger and other Lie transformation groups. The principal aim of the book is the discussion of the symmetry and the construction of exact solutions of specific multidimensional partial differential equations (PDEs), but a number of other problems related to algebraic-theoretic investigations of nonlinear PDEs are also discussed. The results described in the book consist mainly of those obtained by the authors and their collaborators over a period of many years. There is also an English language version of the book published by Reidel at approximately the same time as the present book.

The book stands apart from other books concerned with group properties of differential equations [G. W. Bluman and J. D. Cole, *Similarity methods for differential equations*, Springer, New York, 1974; MR0460846 (57 #838); L. V. Ovsyannikov, *Group analysis of differential equations* (Russian), “Nauka”, Moscow, 1978; MR0511921 (80d:58072); N. Kh. Ibragimov, *Transformation groups in mathematical physics* (Russian), “Nauka”, Moscow, 1983; MR0734307 (85j:58003); P. J. Olver, *Applications of Lie groups to differential equations*, Springer, New York, 1986; MR0836734 (88f:58161); Bluman and S. Kumei, *Symmetries and differential equations*, Springer, New York, 1989; MR1006433 (91b:58284)] by its emphasis on the consideration of equations from quantum theory. A particular interest of the authors is the representation of solutions of nonlinear spinor field equations corresponding to spin one-half since, as is discussed in detail, from these one can construct fields which are solutions of corresponding field equations associated with arbitrary spin.

In Section 1.4, the main section of the book, the authors’ method of reduction of PDEs to ordinary differential equations (ODEs) and the subsequent construction of their exact solutions is described. This method is then applied throughout the remainder of the book to PDEs of various types. The basic idea of the authors’ procedure is the representation of solutions in special forms (ansätze) in which new variables are expressed as linear combinations of the generators of the group with respect to which the given PDE is invariant.

Although the first attempt at a symmetry analysis of a given nonlinear PDE is usually made through the use of Lie’s algorithm, it is now known that not all symmetries are exhausted by local transformations generated by differential operators of first order. Much work on the non-Lie approach in which these first-order operators are replaced by higher-order differential and even integro-differential operators has been done by the authors and their collaborators, and this work is also summarized.

The book consists of five chapters and five appendices. The first two chapters are devoted to the discussion of nonlinear relativistic equations, and systems of such equations, respectively, that are invariant with respect to the Poincaré and conformal groups. Point and tangential symmetries and exact solutions are found for relativistic Hamiltonian and eikonal equations, nonlinear wave equations, Born-Infeld equations, nonlinear Dirac equations, and other types of relativistic quantum field equations. The next two chapters describe a large class of linear and nonlinear PDEs and systems of such equations invariant with respect to the Euclidean, Galilei, and Schrödinger groups. Exact solutions are found for nonlinear heat equations, linear and nonlinear Schrödinger equations, Hamilton-Jacobi equations, nonrelativistic analogues of the Born-Infeld equations, the Boussinesq equations, nonrelativistic linear and nonlinear field equations corresponding to spin one-half, the equations of gas dynamics, and generalized Lamé equations. The final chapter is devoted to some miscellaneous topics including the application of the method of linearization by the use of certain nonlocal transformations obtained from group-theoretic considerations to solve some nonlinear PDEs, including nonlinear wave equations and the Liouville and Monge-Ampère equations. The classical three-body problem is analyzed from the group-theoretic point of view using Lie's algorithm. It is pointed out how the class of nonlinear linearizations is essentially extended by the use of nonlocal transformations.

I was very impressed with this book. With its discussion of many linear and nonlinear partial differential equations of mathematical physics, especially the discussion of non-Lie symmetry methods, one can find many topics here that cannot be found in books of similar type. I strongly recommend it, especially for students and other researchers who are beginning their study of the subject.

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Current version of review. Go to earlier version.

Reviewed by *Woodford W. Zachary*