Assignment 4

Turn in starred problems Wednesday, February 15, at the beginning of the period.

Special instructions: Please write out problems 2.4.7 and 4.B separately from the remaining problems, 2.3.13, 2.4.6, 2.5.5, and 4.A. Do not staple the two different sets of problems together; If you need more than one page for either set, staple them separately.

Exercises from Abbott, *Understanding Analysis*:

Section 2.3: 13*

Section 2.4: 3, 6*, 7*

Section 2.5: 1, 2, 5*, 9

4.A* Define the sequence (c_n) by $c_1 = 2$, $c_{n+1} = 2/(7 - c_n)$ for $n \geq 1$.

(a) Prove that $c_n^2 - 7c_n + 2 < 0$ for all n.

(b) Prove that (c_n) converges, and find its limit.

4.B* Let (a_n) be a bounded sequence of real numbers, and let A be the set of all real numbers which are limits of subsequences of (a_n):

$$A = \left\{ a \in \mathbb{R} \mid a = \lim_{k\to\infty} a_{n_k} \text{ for some subsequence } (a_{n_k}) \text{ of } (a_n) \right\}.$$

Prove that $\sup A$ exists and that $\limsup a_n = \sup A$. (See Exercise 2.4.7 for the definition of $\limsup a_n$. It is also true that $\liminf a_n = \inf A$, but you need not prove this.)