Assignment 5

Turn in starred problems Wednesday, February 22, at the beginning of the period. See the remarks below for hints or modifications of several of these problems.

Exercises from Abbott, *Understanding Analysis*:

Section 2.4: 10*

Section 2.7: 2, 4*, 6, 8*, 9, 10*

Comments, hints and instructions:

2.4.10, 2.7.10: In his discussion of infinite products, Abbott is off the mark in one important respect. The standard convention is that the infinite product $\prod_{k=1}^{\infty} b_k$ converges if the limit of the partial products, $\lim_{n\to\infty} \prod_{k=1}^{n} b_k$, exists *and is nonzero*. An infinite product cannot "converge to 0," as Abbott suggests in 2.7.10(b); such a product *diverges*.

2.4.10: You may use the given inequality, $1 + x \leq 3^x$ for $x \geq 0$, without proof. The better inequality $1 + x \leq e^x$ actually holds for *all* x.

2.7.10(b): In view of remark above, a correct version of this question would be:

> The partial products of the infinite product $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \frac{9}{10} \cdots$ certainly have a limit. (Why?) Does the infinite product converge?

One approach is to use the inequality mentioned in the comment on 2.4.10 above. Note that the criterion for convergence that was obtained in 2.4.10 does not apply here, since there the a_n were nonnegative.