Assignment 8

Turn in starred problems Wednesday, March 29, at the beginning of the period. See the remarks below for hints or modifications of several of these problems.

Exercises from Abbott, *Understanding Analysis*:

Section 4.2: 1(c)*, 5(c,d)*, 6, 7, 8, 9*. 10(b), 11

Section 4.3: 1, 2, 6, 8, 9*, 12*

8.A* Suppose that $A \subset \mathbb{R}$, that $c \in A$, that $f, g : A \to \mathbb{R}$ are continuous at c, and that $f(c) > g(c)$. Prove that there exists a $\delta > 0$ such that $f(x) > g(x)$ for all $x \in A \cap V_\delta(c)$.

Optional extra credit problem; turn in in lecture Thursday 3/30: Abbott 4.3.14. For an extra credit problem, please to not consult any sources or work with other students.

Comments, hints and instructions:

4.2.9: The concepts of infinite limits and limits at infinty are important, and Abbott is not too careful in stating them here. Specifically:

- In the given definition, the domain A of f should be mentioned and c should be a limit point of A; see Definition 4.2.1. Note that Abbott follows his convention of omitting to specify that $x \in A$, even when this is necessary for some statement to make sense.
- In (b) your definition should include some statement which corresponds to the requirement in Definition 4.2.1 that c be a limit point of A. The simplest, if not the most general, condition is to assume that for some $a \in \mathbb{R}$, $f(x)$ is defined for all $x > a$.
- For all the parts (a)–(c) there are corresponding statements with ∞ replaced by $-\infty$: $\lim_x f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = L$, etc. Think about how these should be phrased, but son't write them out to turn in.

4.2.10: Part (a) was included with Workshop 8.

4.2.11: As in 4.2.1 you should be able to prove a theorem like this either directly from Definition 4.2.1 or via the squeeze theorem for sequences, using Theorem 4.2.3.