Assignment 10

Turn in starred problems Wednesday, April 19, at the beginning of the period. See the remarks below for hints or modifications of several of these problems.

Exercises from Abbott, *Understanding Analysis*:

Section 4.6: 1, 2*, 6

Section 5.2: 2*, 5*, 7, 8, 9*, 10, 12*

10.A (Extra credit; turn in (b) in lecture 4/20 if you do it.) For n a nonnegative integer define

$$f_n(x) = \begin{cases} x^n \sin(1/x), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Let $m = \lfloor n/2 \rfloor = \max\{i \in \mathbb{Z} \mid i \leq n/2\}$. We want to prove the

Theorem: f_n is m-times, but not $(m+1)$-times, differentiable. The m^{th} derivative is discontinuous if n is even and continuous if n is odd. (Note: by convention, the 0^{th} derivative of a function g is g itself: $g^{(0)} = g$.)

(a) Review our work in class verifying the theorem for $n = 0$, 1, and 2,

(b)* Prove the theorem. Remember that to find $f_n^{(k)}(0)$ (or show that it does not exist) you must use the definition of derivative as a limit of a difference quotient.

Comments, hints and instructions:

4.6.6: We proved this result in a different way in class on April 3.

5.2.5: I think Abbott intends a to be an arbitrary real number here, but unfortunately we have not defined x^a for general real a. However, in doing the problem you may ignore this difficulty and assume that if $a \in \mathbb{R}$ then x^a is defined for $x > 0$, and for $x = 0$ when $a \geq 0$, and that if $f(x) = x^a$ then $f'(x) = ax^{a-1}$. (Recall that when $a \in \mathbb{Q}$ we did all this fairly carefully in class on April 10).

5.2.7 The case in which a is a nonnegative integer was discussed in class and is further explored in Exercise 10.A above.

5.2.12 The formula for $(f^{-1})'$ is an easy consequence of the chain rule, but proving that this derivative exists is a little more subtle.