WRITING PROOFS

Most of the homework, workshop, and exam problems in this course will ask you to write proofs; your work will be graded carefully for both content and style. We will be talking about how to write proofs throughout the course, but here are some general guidelines.

Content—the logic of the proof

- A proof should contain a clear and rigorous chain of reasoning leading from some given hypotheses, and the definitions of the concepts involved, to the conclusion. You should give arguments supporting all the steps. (More precisely, give arguments for all the steps except for those which are "obvious". Learning what this means—i.e., learning how much to write down for each argument—is part of learning the art of writing proofs). Everything written down should be relevant to this chain of reasoning: don't start by writing down a list of things you know, and don't digress as you go along.
- How does one find the chain of reasoning which makes up the proof? There are no universal answers, but here are a few hints.
  - Begin by establishing clearly in your mind *what you know* and *what you want to prove.*
  - *Know and use the definitions of all the concepts involved.* For many simple proofs, the logic needed is almost forced on you by the definitions you must use.
  - *Use scratch paper.* There you can play with various ideas and, through these, understand the logic of your proof clearly before you begin to write it up for someone else to read.
- A common student error in writing a proof is to work backward from the desired conclusion to the given hypotheses. This may be helpful at the preliminary stage of figuring out how to construct a proof, but is never correct for the final version; such work belongs only on scratch paper. Once you have found the logic by working backwards you must rewrite the proof, moving from what you know to what you want to establish, to be sure that the logic works in that direction. Another way to say this is that in a proof you should never write down a statement or

equality unless you *know* that it is true—from definitions or previously established results, from the hypotheses which are given, or from some chain of reasoning based on these—or unless you state explicitly that it has *not* been established. (For example, you might begin "We must show that ...," or a proof by contradiction might begin "We proceed by contradiction; suppose then that such-and-such is true.")

- When you are done, read your proof critically. Pretend that it was written by a stranger, and that you did not know what he or she was thinking. Does the proof then convince you absolutely? If not, try again.

Style—the language of the proof

- Write in complete, grammatically proper sentences. Remember that the equal sign is a verb. Avoid dangling modifiers.
- Study proofs in the text, in other books, and from the lectures, to get a feeling for good mathematical style. Be aware, however, that the proofs in our text, by Abbott, are for pedagogical reasons frequently much more discursive than your own proofs should be.
- Don't use the notations $\forall$ and $\exists$. Don't introduce mysterious abbreviations to save writing out words. You are allowed to use the abbreviation "iff" for "if and only if".
- Read the document *Writing Proofs*, by Professor James Munkres of MIT, which is posted on the class web page.

Homework and workshop write-ups—when you have time to do it right

- *Start the homework early!* Then, if you just don't know how to get started or you get lost in the middle, talk to me, to our workshop instructor Semeon Artamonov, or to other students.
- Don't turn in scratch work. Once you have decided how to construct a proof, and written out the details once, rewrite the proof neatly to turn in.
- When your work is returned to you, read the comments and be sure you understand their point. If you don't, come to see me or Semeon to talk about them.