Turn in starred problems Wednesday 111/2/2016. Note that this includes parts (a)–(c) of 8(a).

Section 7.4: 7(a), (b), (c)

Section 7.5: 4*

Section 9.9: 4 (a), (d)*

Section 9.10: 2 (a), (c)*; 3

8.A Two interacting populations $x(t), y(t)$ are described by the equations

$$x' = (3 - x - y)x \; , \qquad y' = (x - 2)y \; .$$

(a)* Find all the critical points of this system, and the type of each. **You do not need to do more than this, e.g., you are not asked to find the eigenvalues, etc.**

(b)* Sketch the first quadrant $x \geq 0$, $y \geq 0$ of the phase plane, indicating, by arrows or otherwise, regions where x and y are increasing, x is increasing and y decreasing, etc., and where the trajectories are horizontal and vertical.

(c)* For each initial condition below, find (from your sketch or otherwise) $\lim_{t\to\infty} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$. Explain your reasoning.

(i) $x(0) = 0, \quad y(0) = 3;$ (ii) $x(0) = 3, \quad y(0) = 3;$ (iii) $x(0) = 1, \quad y(0) = 0.$

(d) Give an ecological interpretation of the model, interpreting each term on the right hand side of the differential equations. This a predator-prey model; how would you interpret its difference from the Lotka-Volterra model we discussed in lecture?

8.B* Exercise 1 from Section 1.7 of the notes on Expansions in Orthogonal Bases, available on the web page.

Comments: (a) For the problems in Section 9.10: the best approximation to a given vector within the "span" of some vectors $\{\mathbf{e}_1, \mathbf{e}_2, \ldots\}$ means the best approximation as a linear combination of those vectors. Recall formula (1.24) of the posted lecture notes?

(b) Exercise 8.B involves the evaluation of many integrals; these are simple but can be time-consuming. **You are welcome to use Maple, Mathematica, or some other program to do these.** If you do so, write the integral out out explicitly before giving the answer.