Turn in starred problems Wednesday 11/30/2016.

Section 17.7: 7

Section 17.8: 2 (b)*, (d)*, 5

13.A* Here is a variant of the periodic boundary condition problem of Section 17.8: Find the eigenvalues and eigenfunctions for

$$y'' + \lambda y = 0, \quad y(0) = -y(1), \quad y'(0) = -y'(1).$$

Find also the eigenfunction expansion of $f(x) = 1$.

13.B* A metal plate has the form of a quarter of a disk; it fills the region described in polar coordinates by $0 \le r \le a$, $0 \le \theta \le \pi/2$. Heat conduction in this plate, with Dirichlet boundary conditions on all edges, is described by the equations:

$$\begin{gathered}
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}, \qquad 0 \le r \le a, \quad 0 \le \theta \le \pi/2, \quad t > 0; \\
u(a, \theta, t) = 0, \qquad 0 \le \theta \le \pi/2, \quad t > 0; \\
u(r, 0, t) = u(r, \pi/2, t) = 0, \qquad 0 \le r \le a, \quad t > 0;
\end{gathered}$$

Use separation of variables to find all product solutions $u(r, \theta, t) = R(r)\Theta(\theta)T(t)$ of these equations. Follow the model of the solution for the full disk in the posted notes. Note that you are not asked to form superpositions of these product solutions or to solve any initial value problem.

Hints and remarks:

1. 17.7:7: This shows that innocent looking but nonseparated boundary conditions can lead to trouble.

2. Exercise 17.8:2(d): This is the Legendre equation that we studied earlier (Section 4.4). The requirement that the solution be bounded at $x = 1$ requires that it be one of the Legendre polynomials (see problem 3.A on Assignment 3); you may use this fact without proving it. The boundary condition at $x = 0$ picks out some of these. (These two considerations together determine the eigenvalues.) The text solution for 17.8:2(e) may be helpful. 17.8:2(g) is similar.

3. 17.8:5: This is essentially the problem we encountered in studying the heat equation in a disk: a change of variables leads to Bessel's equation.

4. In 13.B, separation of variables should lead to a problem for $\Theta(\theta)$ which is familiar; you don't have to rederive its solution. In solving the Sturm-Liouville problem for $R(r)$ you may as needed make convenient assumptions about the sign of the eigenvalues, as in the notes.