This assignment is for your benefit only; none of the problems will be collected.

Problems on the Fourier transform, the wave equation, and Laplace's equation:

Section 17.10: 2, 3*, 4 (c), (f), 6 (a), (c), (g)

Section 18.4: 1, 6, 8(a), (b)

Section 19.4: 4, 6 (a), (c)

Section 20.2: 1 (b), (c), 3(e) (without the plots)

Comments, hints, instructions: 1. Hint for 17.10:3: Treat separately the integrals over $x > 0$ and $x < 0$.

2. In 18.4:8(b) one should use the Laplace transform (in the variable t), not the Fourier transform, although in fact one can guess the form of the solution, and then find it completely, by elementary reasoning.

3. Noticed that the text's hints for 19.2.8 follow exactly the method for inhomogeneous problems that we have discussed: y_p is a particular solution depending only on x.

4. In 19.4.6, follow modified instructions:

(a) Find formulas for $u(x, t)$ for all $t > 0$, from d'Alembert's solution (17). This is a bit awkward because one need different formulas in different regions of x-t space.

(b) Don't try for the tricky graphs that Greenberg requests, unless you are very good at sketching. Rather, draw graphs of $u(x.t)$, as a function of x, for various fixed values of t. To really see what is going on, use $t = 0.05$, $t = 0.1$, $t = 0.2$, and $t = 0.4$.

Review problem:

14.A Consider the following problem for the function $u(x, t)$:

$$4\,u_{xx} = u_t, \qquad 0 < x < 3, \quad t > 0; \tag{14.1}$$

$$u(0, t) + a u_x(0, t) = 0, \qquad u(3, t) = 0, \qquad t > 0; \tag{14.2}$$

$$u(x, 0) = 1, \qquad 0 < x < 3. \tag{14.3}$$

(a) Separate variables and investigate the eigenvalues of the resulting Sturm-Liouville problem. In particular, show that (i) if $a > 3$ or $a \le 0$ then all eigenvalues are positive, (ii) if $a = 3$ then zero is an eigenvalue and all other eigenvalues are positive, and (iii) if $0 < a < 3$ then there is one negative eigenvalue and all other eigenvalues are positive. You will not be able to find the eigenvalues analytically (except when $a = 0$).

(b) In each case above, find the solution of the problem as an infinite series. Express the coefficients as ratios of integrals, but do not attempt to evaluate them. The series and integrals will involve the eigenvalues from (a), so you won't be able to be too specific.

(c) Discuss the behavior of $u(x, t)$ as $t \to \infty$. You should find, in the various cases of (a), that (i) $u(x, t)$ approaches zero as $t \to \infty$; (ii) $u(x, t)$ approaches a non-zero steady state as $t \to \infty$; (iii) $u(x, t)$ becomes infinite ("blows up") as $t \to \infty$.

(d) What is the physical interpretation of the boundary condition at $x = 0$ when $a > 3$, and why, on physical grounds, does the solution blow up in that case?