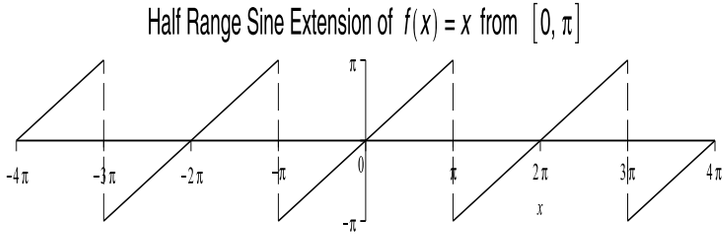
Fractional range sine and cosine series

Note: In each of the four examples that follow—half range sine (HRS) series, half range cosine (HRC) series, quarter range sine (QRS) series, and quarter range cosine (QRC) series—the expansion given for a function $f(x)$ may be thought of as either

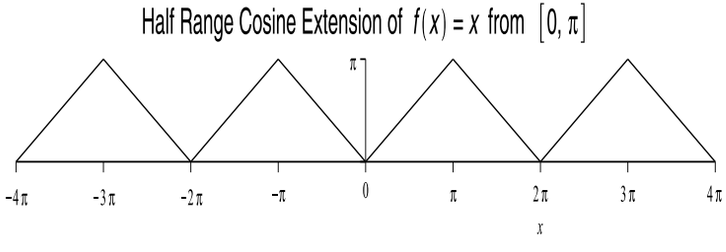
- the expansion of $f(x)$ on $[0, L]$ in terms of the given orthogonal set, or

- the Fourier series of the given periodic extension of $f(x)$.

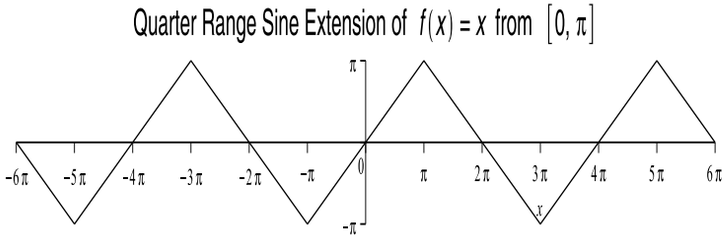
Half range sine (HRS) series

BV problem	$X''(x) + \lambda X(x) = 0, \quad 0 < x < L$ $X(0) = 0, \quad X(L) = 0$
Orthogonal set on $[0, L]$	$\left\{ \sin \dfrac{n\pi x}{L} \mid n = 1, 2, 3 \ldots \right\}$
Expansion of $f(x) \in C_p[0, L]$	$f(x) = \displaystyle\sum_{n=1,2,3,\ldots} b_n \sin \frac{n\pi x}{L}$ $b_n = \dfrac{2}{L} \displaystyle\int_0^L f(x) \sin \frac{n\pi x}{L}\, dx$
Relevant periodic extension $f_1(x)$ of $f(x) = x$ (figure drawn for $L = \pi$)	

Half range cosine (HRC) series

BV problem	$X''(x) + \lambda X(x) = 0, \quad 0 < x < L$ $X'(0) = 0, \quad X'(L) = 0$
Orthogonal set on $[0, L]$	$\{1\} \cup \left\{ \cos \dfrac{n\pi x}{L} \mid n = 1, 2, 3 \ldots \right\}$
Expansion of $f(x) \in C_p[0, L]$	$f(x) = a_0 + \sum_{n=1,2,3,\ldots} a_n \cos \dfrac{n\pi x}{L}$ $a_0 = \dfrac{2}{L} \int_0^L f(x)\, dx$ $a_n = \dfrac{2}{L} \int_0^L f(x) \cos \dfrac{n\pi x}{L}\, dx$
Relevant periodic extension $f_2(x)$ of $f(x) = x$ (figure drawn for $L = \pi$)	

Quarter range sine (QRS) series

BV problem	$X''(x) + \lambda X(x) = 0, \quad 0 < x < L$ $X(0) = 0, \quad X'(L) = 0$
Orthogonal set on $[0, L]$	$\left\{ \sin \dfrac{n\pi x}{2L} \mid n = 1, 3, 5 \ldots \right\}$
Expansion of $f(x) \in C_p[0, L]$	$f(x) = \sum_{n=1,3,5,\ldots} b_n \cos \dfrac{n\pi x}{2L}$ $b_n = \dfrac{2}{L} \int_0^L f(x) \sin \dfrac{n\pi x}{2L} \, dx$
Relevant periodic extension $f_3(x)$ of $f(x) = x$ (figure drawn for $L = \pi$)	

Quarter range cosine (QRC) series

BV problem	$X''(x) + \lambda X(x) = 0, \quad 0 < x < L$ $X'(0) = 0, \quad X(L) = 0$
Orthogonal set on $[0, L]$	$\left\{ \cos \dfrac{n\pi x}{2L} \mid n = 1, 3, 5 \ldots \right\}$
Expansion of $f(x) \in C_p[0, L]$	$f(x) = \displaystyle\sum_{n=1,3,5,\ldots} a_n \cos \frac{n\pi x}{2L}$ $a_n = \dfrac{2}{L} \displaystyle\int_0^L f(x) \cos \frac{n\pi x}{2L}\, dx$
Relevant periodic extension $f_4(x)$ of $f(x) = x$ (figure drawn for $L = \pi$)