

These have not been checked carefully.

1. (a)  $Y(s) = (1 + F(s))/(s^2 - 6s + 9)$ ; (b)  $y(t) = te^{3t} + H(t-1)(t-1)e^{3(t-1)}$ .
2. (a) Yes;  $y(x) = 2 - x$ ; (b)  $\tan 2\sqrt{\lambda} = 2\sqrt{\lambda}$ ,  $\phi_n(x) = \sin(\sqrt{\lambda_n}(x-2))$ ;  
 (d)  $f(x) = c_0(2-x) + \sum_{n=1}^{\infty} c_n \sin(\sqrt{\lambda_n}(x-2))$ ,  $c_0 = 3 \int_0^2 f(x)(2-x) dx / 8$ ,  
 $c_n = \int_0^2 f(x) \sin(\sqrt{\lambda_n}(x-2)) dx / \int_0^2 \sin^2(\sqrt{\lambda_n}(x-2)) dx$ .
3. (b)  $u(x,t) = v(x) + \sum_{n \text{ odd}} \cos(n\pi x/2) e^{-n^2 t}$ , with  $a_n = -(2/\pi) \int_0^\pi v(x) \cos(n\pi x/2) dx$   
 and  $v(x) = x^2/8 + x + 1 - \pi - \pi^2/8$ .
4. (a)  $\hat{u}(\omega, t) = C(\omega)e^{-4i\omega t}$ ; (b)  $\hat{u}(\omega, t) = \hat{f}(\omega)e^{-4i\omega t}$ ; (c)  $u(x, t) = f(x - 4t)$ .
5. From Appendix D: (a)  $e^{-ix}e^{-x^2/4}/\sqrt{\pi}$ ; (b)  $2e^{-3i\omega}/(\omega^2 + 1) + 4/(\omega^2 + 4)$ .
6. (b)  $r_1 = 1$ ,  $r_2 = -1/2$ , (b)  $x + x^3/14 + x^5/616 + \dots$ , (c)  $((n+r)^2 - (n+r) - 1)a_n = a_{n-2}$ .
7. (a)  $X'' + \lambda X = 0$ ,  $T'' + (\lambda - 1)T = 0$ .  
 (b)  $u(x, t) = \sum_{n=1}^{\infty} \sin n\pi x (A_n \cos(\omega_n x) + B_n \sin(\omega_n x))$ ,  $\omega_n = \sqrt{n^2\pi^2 - 1}$ .  
 (c)  $A_n = 0$ ,  $B_n = (2/\omega_n) \int_0^1 x \sin(n\pi x) dx$ .
8. (a) At  $(0, 0)$  unstable focus and at  $(\pm 1, 0)$  saddle, in both linearized and true systems.  
 (b) At  $(0, 0)$  clockwise spiral out; at  $(\pm 1, 0)$  straight lines  $y = 2x$  out,  $y = -x$  in.
9. (a)  $u(x, y) = \sum_{n=1,3,5,\dots} (-1)^n \frac{4}{n\pi \sinh(4n\pi/3)} \sin \frac{n\pi y}{3} \sinh \frac{n\pi x}{3}$ ;  
 (b) Correction: the boundary condition in (b) should have been  $u(x, 3) = 2 \sin 3\pi x$ .  
 $u(x, y) = \sum_{n=1,3,5,\dots} (-1)^n \frac{4}{n\pi \sinh(4n\pi/3)} \sin \frac{n\pi y}{3} \sinh \frac{n\pi x}{3} + \frac{2}{\sinh(9\pi y)} \sin 3\pi x \sinh 3\pi y$ ;  
 (c)  $u(x, y) = \frac{1}{\sinh(21\pi/8)} \cos \frac{7\pi x}{8} \sinh \frac{7\pi(3-y)}{8}$ .