Turn in starred problems Tuesday 2/7/2017.

Section 22.2: 1* (but **see instruction 1 below**).

Section 22.3: 10 (a), (e), (g)*, 11 (a), (b)*, 14(a), (d)*, (e)

4.A In each case below, find a conformal mapping $w = f(z)$ carrying the given region D onto the upper half plane $v > 0$ (here $w = u + iv$ and in describing D we always write $z = x + iy$). Give a brief explanation of your answer, but not a full proof.

(a) D is the right half plane $x > 0$.

(b) D is the second quadrant $x < 0$, $y > 0$. Hint: think about z^2.

(c)* D is the intersection of the right half plane with the unit disk: $x > 0$, $x^2 + y^2 < 1$. Hint: start with a bilinear transfomation, then use the idea of (b).

(d)* D is the strip $1 < x < 2$.

(e) D is the half strip $0 < x < 1$, $y > 0$. Hint: modify an earlier homework problem.

4.B* (From Exam 1 2011). Suppose that the function $\psi(x, y)$ is defined for $x > 0$ by $\psi(x, y) = \tan^{-1}(y/x)$, where the value chosen for the inverse tangent is such that $-\pi/2 < \psi(x, y) < \pi/2$.

(a) Show by direct computation that $\psi(x, y)$ is harmonic.

(b) The conclusion in (a) also follows from a general fact about the relation between harmonic and analytic functions. State this fact and explain carefully how it implies that $\psi(x, y)$ is harmonic.

(c) Find a harmonic conjugate $\phi(x, y)$ of $\psi(x, y)$.

Instructions, comments and hints:

1. **Problem 22.2.1(b)** The problem is not well stated; separation of variables is a method of *finding* a solution, but of course you don't need it to show that the *given* $\Psi(u, v)$ is a solution. So proceed as follows:

(b.i) Check that the given Ψ is a solution.

(b.ii) Let $\Phi(u, v) = 10 + 40u$. We know from (b.i) that Φ solves the given problem (since it is obtained from the Ψ given there by setting $A = B = 0$). Show that if $\Psi(u, v)$ is any solution of the original problem (not necessarily the one given in (1.1)) then $\Xi(u, v) = \Psi(u, v) - \Phi(u, v)$ satisfies Laplace's equation *with zero boundary conditions* in the strip.

(b.iii) Use separation of variables to find the general solution of $\Xi(u, v)$ of Laplace's euation with zero boundary conditions (the general solution which can be obtained in this way). Show that combining this with (b.ii) gives a solution $\Psi(u, v)$ of the original problem which generalizes the given one.