These problems will not be collected.

10.A Exercise 5 in the notes on perturbation theory by Prof. Liping Liu, posted on the class web page. However, you need find only the first *two* terms in the expansions.

10.B (a) Find the eigenvalues, and an orthonormal basis of eigenvectors, of the matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}$.

(b) Find the eigenvalues and eigenvectors, to first order in ε, of the matrix $A + \varepsilon B$, where $B = \begin{pmatrix} 3 & 1 \\ 1 & -5 \end{pmatrix}$

10.C Find the eigenvalues and eigenvectors, to first order in ε, of the matrix $A_0 + \varepsilon A_1$, where $A_0 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and $A_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 5 \end{pmatrix}$.

10.D Let D be the set of functions $f(x)$ which are defined and have two derivatives on $[0, L]$ and satisfy $f'(0) = f'(L) = 0$, and let H_ε be the operator which acts on these functions by $H_\varepsilon f = -((1 + \varepsilon x)f')'$.

(a) Show that H_ε is symmetric, that is, that $\langle f, H_\varepsilon g\rangle = \langle H_\varepsilon f, g\rangle$ for functions f and g in D. Here $\langle f, g\rangle = \int_0^L f(x)g(x)\,dx$. Hint: show that $\langle f, Hg\rangle = \int_0^L (1 + \varepsilon x)f'(x)g'(x)\,dx$; this formula may be also useful for computations in (b).

(b) Find all eigenvalues of H_ε to first order in ε. Hint: the eigenvalue problem for H_0 is associated with a "half-range cosine series".

(c) For the smallest eigenvalue of H, find the corresponding eigenvector to first order in ε.