1. $e^{2in}e^{i\ln 5}$, $n = 0, \pm 1, \pm 2, \ldots$

2. (a) $u(x,y) = 1 + \frac{2}{\pi}\tan^{-1}\left(\frac{y}{x-2}\right) - \frac{3}{\pi}\tan^{-1}\left(\frac{y}{x+2}\right)$ with $0 \leq \tan^{-1}(\cdot) \leq \pi$.

(b) $v(x,y) = -\frac{1}{\pi}\ln\bigl(y^2 + (x-2)^2\bigr) + \frac{3}{2\pi}\ln\bigl(y^2 + (x+2)^2\bigr)$

3. (b) The image of D is the complex plane minus cuts along the real axis from 1 to ∞ and -1 to $-\infty$.

4. (a) The portion of the disk $|w| < 4$ wwhich lies in the upper half plane.

(b) One possible answer: $\zeta = \frac{4+w}{4-w} = \frac{4+z^2}{4-z^2}$.

(c) Let $\hat{z} = \zeta^2 = \left(\frac{4+z^2}{4-z^2}\right)^2$. If $\hat{z}(z) = \hat{x}(x,y) + i\hat{y}(x,y)$ then

$$u(x,y) = 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{\hat{y}(x,y)}{\hat{x}(x,y)-1}\right) + \frac{1}{\pi}\tan^{-1}\left(\frac{\hat{y}(x,y)}{\hat{x}(x,y)}\right), \quad 0 \leq \tan^{-1}(\cdot) \leq \pi.$$

5. $\ln(1+\sqrt{3}) + \frac{i\pi}{2} + 2n\pi i$ and $\ln(\sqrt{3}-1) - \frac{i\pi}{2} + 2n\pi i$, $n = 0, \pm 1, \pm 2, \ldots$.

6. Let $\zeta(z) = z + \frac{9}{z} = \xi(x,y) + i\eta(x,y)$ with $\xi(x,y) = \frac{x(x^2+y^2+9)}{x^2+y^2}$ and $\eta(x,y) = \frac{y(x^2+y^2-9)}{x^2+y^2}$; then $u(x,y) = \frac{5}{\pi}\tan^{-1}\left(\frac{\eta(x,y)}{\xi(x,y)-6}\right) - \frac{5}{\pi}\tan^{-1}\left(\frac{\eta(x,y)}{\xi(x,y)+6}\right)$, with $0 \leq \tan^{-1}(\cdot) \leq \pi$.

7. (a) $19/6 - (11/3)i$, (b) $e^2\sqrt{5}/4$.

8. $\sum_{n=0}^{\infty}(z-1)^n/2^{n+1}$, $|z-1| < 2$;

9. $\pi/3$.

10. (a) $\sin'(z) = \cos z$, $\cos n\pi = \pm 1 \neq 0$.

(b) Singularities: $n\pi/4$, $n = 0, \pm 1, \pm 2, \ldots$. Those at $k\pi$ ($n = 4k$, $k = 0, \pm 1, \pm 2, \ldots$) are removable; the others are simple poles.

(c) Zero of order 2 at 0, zeros of order 1 at $k\pi$, $k = \pm 1, \pm 2, \ldots$.

(d) $\sqrt{1 + \pi^2/16}$.

11. $\pi(e^{-2} - e^{-6})/8$.

12. See class notes.